9

Straight Lines

We observe the concept of lines every day in our daily life. When you go to a railway station you can observe the railway tracks. Railway tracks are usually constructed with horizontal supports. Their function is to secure the rail so that they won't shift under the heavy force of the moving trains.

Topic Notes

- *Basics of a Straight Line*
- *Equations of Line*

BASICS OF A STRAIGHT LINE 1

TOPIC 1

SLOPE AND ANGLE BETWEEN TWO LINES

A straight line is a curve such that all the points on the line segment joining any two points on it lies on it.

Slope of a Line

Slope of a line in terms of coordinates of any two points on it.

Let $P(x_1\ y_1)$ and $Q(x_2\ y_2)$ be two points on a line making an angle θ with the positive direction of x-axis

Then its slope is,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a line passing through points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $m = \tan\theta = \left|\frac{y_2 - y_1}{x_2 - x_1}\right|$.

Example 1.1: Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points $P(0, -4)$ and $B(8, 0)$. [NCERT]

Ans. Let M be the mid-point of $P(0, -4)$ and $B(8, 0)$.

We know that

Mid-point of (x_1, y_1) and (x_2, y_2)

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Mid-point M of } (0, -4) \text{ and } (8, 0) = \left(\frac{0+8}{2}, \frac{-4+0}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{-4}{2}\right)$$

$$= (4, -2)$$

We know that slope of the line passing through

(x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope of line between points O(0, 0) and M(4, −2).

Here,

$$x_1 = 0, y_1 = 0$$
$$x_2 = 4, y_2 = -2$$

On putting values, we get

$$m = \frac{-2-0}{4-0}$$

$$= \frac{-2}{4}$$

$$= \frac{-1}{2}$$

Hence, slope of line is $\frac{-1}{2}$.

Example 1.2: Find the slope of the line, making an inclination of 60° with the positive direction of x-axis. [NCERT]

Ans. Given that inclination of a line with the positive direction of x-axis is 60°.

$$\therefore \quad \theta = 60°$$

Hence, the slope of the line,

$$m = \tan\theta$$

$$= \tan 60° = \sqrt{3}$$

Example 1.3: Find the angle between the x-axis and the line joining the points (3, −1) and (4, −2).

Ans. First we find the slope of the line joining the point (3, −1) and (4, −2).

We know that slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here, $x_1 = 3$, $y_1 = -1$ and $x_2 = 4$, $y_2 = -2$

Slope of the joining (3, −1) and (4, −2) is

$$m = \frac{-2-(-1)}{4-3}$$

$$= \frac{-2+1}{4-3}$$

$$= \frac{-1}{1}$$

$$= -1$$

Now,

$$\text{Slope} = m = \tan\theta$$

Where θ is the angle between line and positive x-axis.

So, $\quad m = \tan\theta$

$$-1 = \tan\theta$$
$$\tan\theta = -1$$
$$\tan\theta = \tan(135)°$$
$$\theta = 135°$$

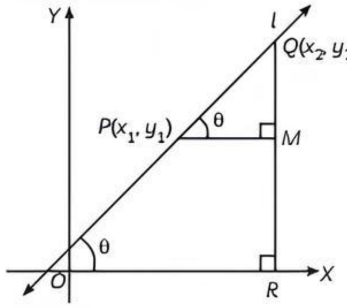
So, required angle = θ = 135°

Angle Between Two Lines

The angle θ between the lines having slope m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

Case I: When angle θ is acute:



In the given figure, $\angle MPQ = \theta$ ...(i)

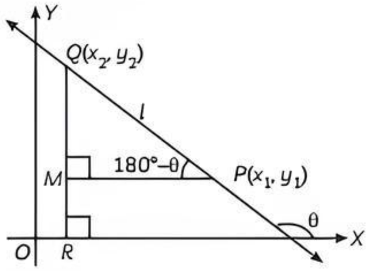
Therefore, slope of line $l = m = \tan\theta$

But in ΔMPQ, we have $\tan\theta = \frac{MQ}{MP} = \frac{y_2 - y_1}{x_2 - x_1}$ ...(ii)

From equations (i) and (ii), we have

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Case II: When angle θ is obtuse:



In the given figure, we have

$\angle MPQ = 180° - \theta$

Therefore, $\theta = 180° - \angle MPQ$

Now, slope of the line l

$$m = \tan\theta$$

$$= \tan(180° - \angle MPQ) = -\tan\angle MPQ$$

$$= -\frac{MQ}{MP} = -\frac{y_2 - y_1}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

Consequently, we see that in both the cases the slope m of the line through the points

(x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

Condition of Parallelism of Lines

If two lines of slope m_1 and m_2 are parallel, then the angle θ between them is 0°.

$$\tan\theta = \tan 0° = 0$$

$$\frac{m_2 - m_1}{1 + m_1 m_2} = 0$$

$$m_2 = m_1$$

Thus, when two lines are parallel, their slopes are equal.

Condition of Perpendicularity of two Lines

If two lines of slopes m_1 and m_2 are perpendicular, then the angle θ between them is 90°.

Hence,

$$m_1 \times m_2 = -1$$

Example 1.4: The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slopes of the lines.

[NCERT]

Ans. Let m_1 and m_2 be the slope of two lines.

We know that angle between two lines is given by

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

Here, $\tan\theta = \frac{1}{3}$ and $m_2 = 2m_1$

On putting values, we get

$$\frac{1}{3} = \left|\frac{2m_1 - m_1}{1 + m_1(2m_1)}\right|$$

$$\frac{1}{3} = \left|\frac{m_1}{1 + 2m_1^2}\right|$$

$$\left|\frac{m_1}{1 + 2m_1^2}\right| = \frac{1}{3}$$

So, $\frac{m_1}{1 + 2m_1^2} = \frac{1}{3}$ or $\frac{m_1}{1 + 2m_1^2} = \frac{-1}{3}$

On solving

$$\frac{m_1}{1 + 2m_1^2} = \frac{1}{3}$$

$$3m_1 = 1 + 2m_1^2$$

$$2m_1^2 + 1 - 3m_1 = 0$$

$$2m_1^2 - 3m_1 + 1 = 0$$

$$2m_1^2 - 2m_1 - m_1 + 1 = 0$$

$$2m_1(m_1 - 1) - 1(m_1 - 1) = 0$$

$$(2m_1 - 1)(m_1 - 1) = 0$$

So, $m_1 = \frac{1}{2}, m_1 = 1$

On solving

$$\frac{m_1}{1 + 2m_1^2} = \frac{-1}{3}$$

$$3m_1 = -1 - 2m_1^2$$

$$2m_1^2 + 1 + 3m_1 = 0$$

$$2m_1^2 + 3m_1 + 1 = 0$$

$$2m_1^2 + 2m_1 + m_1 + 1 = 0$$

$$2m_1(m_1 + 1) + 1(m_1 + 1) = 0$$

$$(2m_1 + 1)(m_1 + 1) = 0$$

So, $m_1 = \frac{-1}{2}, m_1 = -1$

When, $m_1 = \frac{1}{2}$

$m_2 = 2m_1$

$m_2 = 2\left(\frac{1}{2}\right) = 1$

When, $m_1 = 1$

$m_2 = 2m_1$

$m_2 = 2\ (1) = 2$

When, $m_1 = \frac{-1}{2}$

$m_2 = 2m_1$

$m_2 = 2\left(\frac{-1}{2}\right) = -1$

When, $m_1 = -1$

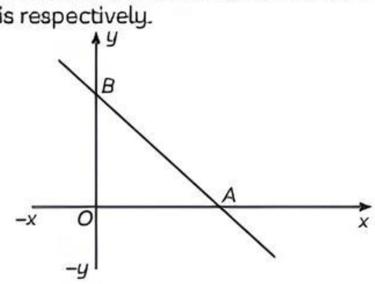
$m_2 = 2m_1$

$m_2 = 2(-1) = -2$

Hence, slope of lines are $\frac{1}{2}$ and 1 or 1 and 2 or $\frac{-1}{2}$ and -1 or -1 and -2.

Intersects of a Line on the Axes

If a straight line cuts x-axis at A and y-axis at B, then OA and OB are known as intersects of the line on x-axis and y-axis respectively.



Point of Intersection of Two Lines

Let the equations of two lines be

$a_1 x + b_1 y + c_1 = 0$ ...(i)

and, $a_2 x + b_2 y + c_2 = 0$ ...(ii)

Suppose these two lines intersect at a point, $P(x_1, y_1)$.

Then, (x_1, y_1) satisfies each of the given equations.

$\therefore \quad a_1 x_1 + b_1 y_1 + c_1 = 0$

and $a_2 x_1 + b_2 y_1 + c_2 = 0$

Solving these two equations by cross-multiplication, we get

$$\frac{x_1}{b_1c_2 - b_2c_1} = \frac{y_1}{c_1a_2 - c_2a_1}$$

$$\Rightarrow \quad x_1 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, y_1 = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Hence, the coordinates of the point of intersection of the line (i) and (ii) are:

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$$

Angle between two Straight Lines When their Equations are given

The acute angle θ between the lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ is given by

$$\tan\theta = \left|\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right|$$

Let m_1 and m_2 be the slope of the lines

$a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$

Then,

$$m_1 = -\frac{a_1}{b_1} \text{ and } m_2 = -\frac{a_2}{b_2}$$

Now, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

$$\Rightarrow \quad \tan\theta = \left|\frac{-\frac{a_1}{b_1} + \frac{a_2}{b_2}}{1 + \left(-\frac{a_1}{b_1}\right)\left(-\frac{a_2}{b_2}\right)}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right|$$

$$\Rightarrow \quad \theta = \tan^{-1}\left|\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right|$$

Condition for the Lines to be Parallel

If the lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel, then

$$m_1 = m_2$$

$$\Rightarrow \quad -\frac{a_1}{b_1} = -\frac{a_2}{b_2}$$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

Condition for the Lines to be Perpendicular

If the lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are perpendiculars, then

$$m_1 m_2 = -1$$

$$\Rightarrow \quad -\frac{a_1}{b_1} \times -\frac{a_2}{b_2} = -1$$

$$a_1a_2 + b_1b_2 = 0$$

If follows from the above discussion that the lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are:

(1) Coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(2) Parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(3) Intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(4) Perpendicular, if $a_1 a_2 + b_1 b_2 = 0$

TOPIC 2

DISTANCE OF A POINT FROM A LINE

The length of the perpendicular from a point (x_1, y_1) to a line $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

Distance between Two Parallel Lines

If two lines are parallel, then the distance between them is the same throughout. Therefore, to find the distance between two parallel lines, choose an arbitrary point on one of them and find the length of the perpendicular on the other. In order to choose a point on a line, we give an arbitrary value to x or y and find the value of the other variable.

We may use the following algorithm to find the distance between two parallel lines.

Let the two parallel lines be, $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$. To find the distance between these two lines, we use following working steps.

Step I: Choose a point on any one of the two lines by giving a particular value to x or y of your choice.

Step II: Find the length of the perpendicular from the chosen point in Step I to the other line.

Step III: The length obtained in Step II is the required distance between the parallel lines.

The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.

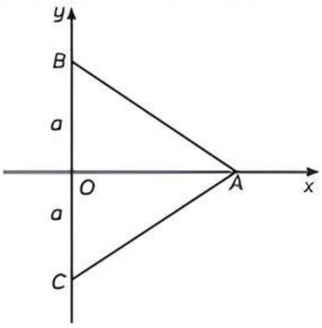
Example 1.5: The base of an equilateral triangle with side $2a$ lies along with the y-axis such that the mid-point of the base is at the origin. Find the vertices of the triangle. [NCERT]

Ans. Let ΔABC be the equilateral triangle with side $2a$ such that its base BC is along the y-axis and the mid-point of the base is at the origin O.

Then, point A lies on the x-axis and OB = OC = a.

The coordinates of points B and C are $(0, a)$ and $(0, -a)$ respectively.

Let $(x, 0)$ be the coordinates of point A.



Since, $AB = 2a$

$\Rightarrow \sqrt{(0-x)^2 + (a-0)^2} = 2a$

$\Rightarrow \sqrt{x^2 + a^2} = 2a$

$\Rightarrow x^2 + a^2 = 4a^2$

$\Rightarrow x^2 = 3a^2$

$\Rightarrow x = \pm\sqrt{3}a$

So, the coordinates of point A are $(\sqrt{3}a, 0)$ or $(-\sqrt{3}a, 0)$.

Hence, vertices of the triangle are $(\sqrt{3}a, 0)$, $(0, a)$, $(0, -a)$ or $(-\sqrt{3}a, 0)$, $(0, a)$, $(0, -a)$.

Example 1.6: Find a point on the x-axis, which is equidistant from the point (7, 6) and (3, 4). [NCERT]

Ans. Point on x-axis $(a, 0)$.

Let $P(a, 0)$, $A(7, 6)$ and $B(3, 4)$

Here, PA = PB

$$\sqrt{(7-a)^2 + (6-0)^2} = \sqrt{(3-a)^2 + (4-0)^2}$$

$$(7-a)^2 + 36 = (3-a)^2 + 16$$

$$49 + a^2 - 14a + 36 = 9 + a^2 - 6a + 16$$

$$49 - 16 + 36 - 9 = 14a - 6a$$

$$60 = 8a$$

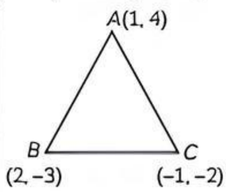
$$\frac{60}{8} = a$$

$$a = \frac{15}{2}$$

So, point is $\left(\frac{15}{2}, 0\right)$.

Example 1.7: Case Based:

Truss bridges are formed with a structure of connected elements that form triangular structure to make up the bridge. Trusses are the triangles that connect to the top and bottom cord and two endposts. Consider the ΔABC with vertices $A(1, 4)$, $B(2, -3)$ and $C(-1, -2)$.



(A) Find the slope of BC.

(B) Find the slope of AC.

(C) Distance between A and C is:

(a) $\sqrt{10}$ (b) $2\sqrt{10}$

(c) $3\sqrt{10}$ (d) $4\sqrt{10}$

(D) Distance between B and C is:

(a) $\sqrt{5}$ (b) $\sqrt{7}$

(c) $\sqrt{9}$ (d) $\sqrt{10}$

(E) Assertion (A): A point $P(h, k)$ lies on the straight line $x + y + 1 = 0$ and is at a distance 5 units from the origin. If k is negative, then h is equal to -3.

Reason (R): The distance formula is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Ans. (A) Here, B(2, – 3) and C(– 1, – 2)

So, Slope of BC is $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-2+3}{-1-2} = \frac{1}{-3} = \frac{-1}{3}$$

(B) Here, A(1, 4) and C(–1, –2).

So, Slope of AC is $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-2-4}{-1-1} = \frac{-6}{-2} = 3$$

(C) *(b)* $2\sqrt{10}$

Explanation: Here, A(1, 4) and C(–1, –2).

So, $$AC = \sqrt{(-1-1)^2 + (-2-4)^2}$$
$$= \sqrt{4+36}$$
$$= \sqrt{40}$$
$$= 2\sqrt{10}$$

(D) *(d)* $\sqrt{10}$

Explanation: Here, B(2, –3) and C(–1, –2)

So, $$BC = \sqrt{(-1-2)^2 + (-2+3)^2}$$
$$= \sqrt{9+1}$$
$$= \sqrt{10}$$

(E) *(d) (A) is false but (R) is true.*

Explanation: Since, the point (h, k) lies on $x + y + 1 = 0$.

$\Rightarrow$ $h + k + 1 = 0$

and $h^2 + k^2 = 25$

$\Rightarrow$ $(-1 - k)^2 + k^2 = 25$

$\Rightarrow$ $2k^2 + 2k - 24 = 0$

$\Rightarrow$ $k^2 + k - 12 = 0$

$\Rightarrow$ $k = -4$ or $k = 3$

[$k = 3$ rejected as $k < 0$]

$\therefore$ $h = -1 - 1(-4) = 3$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. The point (–1. 5, 2. 56) lies in which quadrant?

(a) I (b) II

(c) III (d) IV

Ans. *(b) II*

Explanation: Here, value of x is negative and value of y is positive.

In quadrant II, x is negative and y is positive.

2. The inclination of the line $x - y + 3 = 0$ with the positive direction of x-axis is:

(a) 45° (b) 135°

(c) –45° (d) –135°

[Delhi Gov. Term-1 SQP 2021]

Ans. *(a) 45°*

Explanation: The equation of the line $x - y + 3 = 0$ can be rewritten as

$$y = x + 3$$

Comparing with $y = mx + c$

$\Rightarrow$ $m = \tan\theta = 1$

Hence $\theta = 45°$.

3. The value of y, if the distance between points P(0, 0)and Q(6, y) is 10 units is:

(a) 3, –3 (b) 8, –8

(c) 0 (d) none

Ans. *(b) 8, –8*

Explanation: Given points are P(0, 0) and Q(6, y)

$$PQ = 10$$

$$\sqrt{(6-0)^2+(y-0)^2} = 10$$

$$36 + y^2 = 100$$

[Squaring on both sides]

$$y^2 = 64$$

$$y = 8, -8$$

4. **The point on x-axis which is equidistant from the point (3, 2)and (–5, –2) is:**

(a) (1, 0) (b) (2, 0)

(c) (–1, 0) (d) (–2, 0)

Ans. *(c) (–1, 0)*

Explanation: Let A(3, 2) and B(–5, –2) be the given points and P(x, 0) is the point on x-axis.

$$PA = PB$$

$$\Rightarrow \quad PA^2 = PB^2$$

$$(3-x)^2 + (2-0)^2 = (-5-x)^2 + (-2-0)^2$$

$$9 + x^2 - 6x + 4 = 25 + x^2 + 10x + 4$$

$$16x + 16 = 0$$

$$x = -1$$

Since, the point is on x – axis, so $y = 0$

Thus, the point is (–1, 0).

5. **Line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24). Find the value of x.**

(a) 2 (b) 3

(c) 4 (d) 5

[Delhi Gov. Term-1 SQP 2021]

Ans. *(c) 4*

Explanation: Let points be A(–2, 6), B(4, 8), C(8, 12) and D(x, 24)

If two lines are perpendicular, then product of their slope is –1

So, Slope of AB × Slope of CD = –1 ...(i)

We know that slope of a line through the points (x_1, y_1), (x_1, y_1) is $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope of line AB passing through A(–2, 6) and B(4, 8) is

$$= \frac{8-6}{4-(-2)} = \frac{8-6}{4+2} = \frac{2}{6} = \frac{1}{3}$$

Slope of line CD passing through C(8, 12) and D(x, 4) is

$$= \frac{24-12}{x-8} = \frac{12}{x-8}$$

From (i)

Slope of AB × Slope of CD = – 1

$$\Rightarrow \quad \frac{1}{3} \times \frac{12}{x-8} = -1$$

$$\Rightarrow \quad \frac{4}{x-8} = -1$$

$$\Rightarrow \quad 4 = -x + 8$$

$$\therefore \quad x = 4$$

6. **The tangent of angle between the lines whose intercepts on the axes are a, $-b$ and b, $-a$, respectively, is:**

(a) $\frac{a^2-b^2}{ab}$ (b) $\frac{b^2-a^2}{2}$

(c) $\frac{b^2-a^2}{2ab}$ (d) none of these

[NCERT Exemplar]

Ans. (c) $\frac{b^2-a^2}{2ab}$

Explanation: Let the first equation of line having intercepts on the axes a and $-b$ is

$$\therefore \quad \frac{x}{a} + \frac{y}{-b} = 1$$

$$\Rightarrow \quad \frac{x}{a} - \frac{y}{b} = 1$$

$$\Rightarrow \quad bx - ay = ab \quad ...(i)$$

Let the second equation of line having intercepts on the axes b and $-a$ is

$$\frac{x}{b} + \frac{y}{-a} = 1$$

$$\Rightarrow \quad \frac{x}{b} - \frac{y}{a} = 1$$

$$\Rightarrow \quad ax - by = ab \quad ...(ii)$$

Now, we find the slope of equation (i)

$$bx - ay = ab$$

$$\Rightarrow \quad ay = bx - ab$$

$$\Rightarrow \quad y = \frac{b}{a}x - b$$

Since, the above equation is in $y = mx + b$ form.

So, the slope of equation (i) is

$$m_1 = \frac{b}{a}$$

Now, we find the slope of equation (ii).

$$ax - by = ab$$

$$\Rightarrow \quad by = ax - ab$$

$$\Rightarrow \quad y = \frac{a}{b}x - a$$

Since, the above equation is in $y = mx + b$ form.

So, the slope of eq. (i) is

$$m_2 = \frac{a}{b}$$

Let θ be the angle between the given two lines.

$$\tan\theta = \left|\frac{m_1 - m_1}{1 + m_1 m_2}\right|$$

Putting the values of m_1 and m_2 in the above equation, we get

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{b}{a} - \frac{a}{b}}{1 + \left(\frac{b}{a}\right)\left(\frac{a}{b}\right)}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{b^2 - a^2}{ab}}{1+1}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{b^2 - a^2}{2ab}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{b^2 - a^2}{2ab}$$

7. If the vertices (1, 3), (2, 5) and (3, –5) of a triangle, the centroid is:

(a) (1, 2) **(b) (1, 3)**

(c) (3, 1) **(d) (2, 1)**

Ans. *(d) (2, 1)*

Explanation: Let three points be

$A(x_1, y_1) = (1, 3)$

$B(x_2, y_2) = (2, 5)$

and $C(x_3, y_3) = (3, -5)$

$$\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{1+2+3}{3}, \frac{3+5-5}{3}\right)$$

$$= \left(\frac{6}{3}, \frac{3}{3}\right)$$

$$= (2, 1)$$

8. The line $(3x - y + 5) + \lambda(2x - 3y - 4) = 0$ will be parallel to y-axis, if λ =

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$

(c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

[Delhi Gov. Term-1 SQP 2021]

Ans. *(b)* $\frac{-1}{3}$

Explanation: The given equation is

$(3x - y + 5) + \lambda(2x - 3y - 4) = 0$

$\Rightarrow \quad 3x - y + 5 + 2\lambda x - 3\lambda y - 4\lambda = 0$

$\Rightarrow \quad x(3 + 2\lambda) - y(1 + 3\lambda) + 5 - 4\lambda = 0$

Here sope of the line is $m = \frac{3+2\lambda}{1+3\lambda}$

Since the line is parallel to y-axis, slope is undefined.

$\Rightarrow \quad 1 + 3\lambda = 0$

$\Rightarrow \quad \lambda = -\frac{1}{3}$

9. The slope of the lines which make an angle 45° with the line $3x - y + 5 = 0$ is:

(a) $-2, \frac{1}{2}$ (b) $\frac{1}{2}, -3$

(c) $2, \frac{-1}{2}$ (d) $-3, \frac{1}{2}$ **[Diksha]**

Ans. *(a)* $-2, \frac{1}{2}$

Explanation: Given line is

$3x - y + 5 = 0$

$y = 3x + 5$

On comparing it with $y = mx + b$, we get

$m = 3$

Slope of the line $3x - y + 5 = 0$ is 3.

$$\tan 45° = \left|\frac{m-3}{1+3m}\right|$$

$$1 = \left|\frac{m-3}{1+3m}\right|$$

$|1 + 3m| = |(m - 3)|$

$1 + 3m = \pm (m - 3)$

$m = -2$ or, $m = \frac{1}{2}$

10. What is the angle between the two straight lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$?

(a) 60° **(b) 45°**

(c) 30° **(d) 15°**

[Delhi Gov. Term-1 SQP 2021]

Ans. *(a) 60°*

Explanation: Given lines are $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

From given straight lines $m_1 = 2 - \sqrt{3}$ and $m_2 = 2 + \sqrt{3}$

Substitute above values in tan θ, we get

$$\tan\theta = \left|\frac{2 + \sqrt{3} - 2 + \sqrt{3}}{1 + (4 - 3)}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{2\sqrt{3}}{2}\right|$$

$$\Rightarrow \quad \theta = \tan^{-1}\sqrt{3}$$

Therefore angle is θ = 60°.

11. The area of a triangle whose vertices are (2, 4), (4, –2), and (–3, 10) is:

(a) 4 (b) 9

(c) 3 (d) 10

Ans. *(b) 9*

Explanation: Let three points be

$A(x_1, y_1) = (2, 4)$

$A(x_2, y_2) = (4, -2)$

and $C(x_3, y_3) = (-3, 10)$

Area of triangle

$$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$= \frac{1}{2}|2(-2-10)+4(10-4)+(-3)(4+2)|$$

$$= \left|\frac{-24+24-18}{2}\right|$$

$$= \left|\frac{-18}{2}\right| = 9$$

12. The equation of the straight line that passes through the point (3, 4) and perpendicular to the line $3x + 2y + 5 = 0$ is:

(a) $2x + 3y + 6 = 0$ (b) $2x - 3y - 6 = 0$

(c) $2x - 3y + 6 = 0$ (d) $2x + 3y - 6 = 0$

[Delhi Gov. Term-1 SQP 2021]

Ans. *(c) $2x - 3y + 6 = 0$*

Explanation: The equation of a line perpendicular to $3x + 2y + 5 = 0$ is

$$2x - 3y + \lambda = 0 \quad ...(i)$$

This passes through the point (3, 4)

$\therefore\ 3 \times 2 - 3 \times 4 + \lambda = 0$

$\Rightarrow \lambda = 6$ putting $\lambda = 6$ in (i), we get

$2x - 3y + 6 = 0$, which is the required equation.

13. The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$ is:

(a) $\frac{3}{4}$ (b) $\frac{7}{5}$

(c) $\frac{2}{5}$ (d) $\frac{3}{5}$ [Diksha]

Ans. *(c) $\frac{2}{5}$*

Explanation: Given lines are $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$

On comparing it with $ax + by + c = 0$, we get

$a = 3, b = -4, c_1 = 7, c_2 = 5$

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|7-5|}{\sqrt{(3)^2 + (-4)^2}}$$

$$= \frac{2}{\sqrt{9+16}}$$

$$= \frac{2}{\sqrt{25}}$$

$$= \frac{2}{5}$$

14. The angle between x-axis and the line joining the points (3, –1) and (4, –2) is:

(a) 135° (b) 145°

(c) 270° (d) none of these

[Diksha]

Ans. *(a) 135°*

Explanation: The slope of the line joining the points

(3, –1) and (4, –2) is $m = \frac{-2-(-1)}{4-3}$

$= -2 + 1 = -1$

Now, the inclination (θ) of the line joining the points (3, –1) and (4, –2) is given by $\tan\theta = -1$

$\Rightarrow \quad \theta = (90° + 45°) = 135°.$

A slope of the line is an inclination with the x-axis.

Thus, the angle between x – axis and the line joining the points (3,1) and (4,2) is 135°.

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of A.

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

15. Assertion (A): The angle between the line $x + 2y - 3 = 0$ and $3x + y + 1 = 0$ is $\tan^{-1}(1)$.

Reason (R): Angle between two lines is given by $\tan\theta = \pm\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$

Ans. (a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: Let m_1 and m_2 be the slopes of the straight lines $x + 2y - 3 = 0$ and $3x + y + 1 = 0$.

Then, $m_1 = -\frac{1}{2}$ and $m_2 = -3$

Let θ be the angle between the given lines.

Then, $\tan\theta = \pm\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$

$$= \pm\left(\frac{-3+\frac{1}{2}}{1+\frac{3}{2}}\right) = \pm 1$$

$\Rightarrow \quad \theta = \tan^{-1}(1)$

16. Assertion (A): A slope of line $3x - 4y + 10 = 0$ is $\frac{3}{4}$.

Reason (R): x-intercepts and y-intercepts of $3x - 4y + 10 = 0$ respectively are $\frac{-10}{3}$ and $\frac{5}{2}$.

Ans. (b) *Both (A) and (R) are true but (R) is not the correct explanation of (A).*

Explanation: Given equation $3x - 4y + 10 = 0$ can be written as

$$y = \frac{3}{4}x + \frac{5}{2} \quad \text{...(i)}$$

Comparing eq. (i) with $y = mx + c$, we have slope of the given line as $m = \frac{3}{4}$.

Equation $3x - 4y + 10 = 0$ can be written as

$$3x - 4y = -10 \text{ or } \frac{x}{-\frac{10}{3}} + \frac{y}{\frac{5}{2}} = 1 \quad \text{...(ii)}$$

Comparing eq. (ii) with $\frac{x}{a} + \frac{y}{b} = 1$, we have x-intercept as $a = -\frac{10}{3}$ and y-intersect as $b = \frac{5}{2}$.

17. Assertion (A): If $x \cos\theta + y \sin\theta = 2$ is perpendicular to the line $x - y = 3$ then one of the values of θ is $\frac{\pi}{4}$.

Reason (R): If two lines $y = m_1 x + c_1$ and $y = m_2 x + c_2$ are perpendicular then $m_1 = m_2$.

Ans. (c) *(A) is true but (R) is false.*

Explanation: Since, slope of line $x \cos\theta + y \sin\theta = 2$ is $-\cot\theta$ and slope of line $x - y = 3$ is 1.

Also, these lines are perpendicular to each other.

$\therefore \quad (-\cot\theta)(1) = -1$

$\Rightarrow \quad \cot\theta = 1$

$\Rightarrow \quad \theta = \cot^{-1}\left(\cot\frac{\pi}{4}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

Condition of perpendicularity of two lines is $m_1 \cdot m_2 = -1$ and not $m_1 = m_2$.

18. Assertion (A): The slope of the line $x + 7y = 0$ is $\frac{1}{5}$ and y-intercept is 0.

Reason (R): The slope of the line $6x + 3y - 5 = 0$ is -2 and y – intercept is $\frac{5}{2}$.

Ans. (d) *(A) is false but (R) is true.*

Explanation: Given equation is $x + 7y = 0$

$\Rightarrow \quad y = \frac{-x}{7} + 0$

On comparing with $y = mx + c$, we get

Slope $(m) = \frac{-1}{7}$, y – intercept = 0

Given equation is $6x + 3y - 5 = 0$

$\Rightarrow \quad y = -2x + \frac{5}{3}$

On comparing with $y = mx + c$, we get

Slope $(m) = -2$, y – intercept = $\frac{5}{3}$

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

19. *If A and B are two persons standing at the positions (2, –3) and (6, –5). If C is a third person who is standing between A and B such that it divides the line AB in the ratio 1 : 3.*

Based on the above information answer the following questions.

(A) The distance between A and B is:

(a) $\sqrt{5}$ (b) $2\sqrt{5}$

(c) $3\sqrt{5}$ (d) $4\sqrt{5}$

(B) The equation of AB is:

(a) $x + 2y + 4 = 0$

(b) $x + 2y - 4 = 0$

(c) $x - 2y + 4 = 0$

(d) none of these

(C) Coordinates of points C are:

(a) $\left(\frac{7}{2}, -3\right)$ (b) $\left(3, \frac{7}{2}\right)$

(c) (3, 3) (d) $\left(3, -\frac{7}{2}\right)$

(D) Distance between A and C is:

(a) $\sqrt{5}$ (b) $2\sqrt{5}$

(c) $\frac{\sqrt{5}}{2}$ (d) $\sqrt{\frac{5}{2}}$

(E) Distance between C and B is:

(a) $\frac{3\sqrt{5}}{2}$ (b) $3\sqrt{5}$

(c) $\frac{2\sqrt{5}}{3}$ (d) None of these

Ans. (A) *(b)* $2\sqrt{5}$

Explanation: Given positions of person A and B are as follows:

A(2, –3) and B(6, –5)

$$d = \sqrt{(6-2)^2 + (-5+3)^2}$$

[using distance formula]

$$= \sqrt{(4)^2 + (-2)^2} = \sqrt{16+4}$$

$$= \sqrt{20} = 2\sqrt{5}$$

(B) *(a)* $x + 2y + 4 = 0$

Explanation: We have, A(2, –3) and B(6, –5)

$$\text{Slope, } m = \frac{-5-(-3)}{6-2}$$

$$= \frac{-5+3}{4}$$

$$= \frac{-2}{4} = -\frac{1}{2}$$

Taking point A(2, –3) = (x_1, y_1) and $m = -\frac{1}{2}$

Equation of line AB is

$$(y - (-3)) = -\frac{1}{2}(x-2)$$

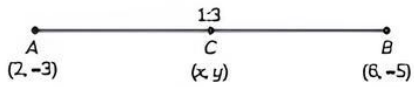
$\Rightarrow$ $2(y + 3) = -(x - 2)$

$\Rightarrow$ $2y + 6 = -x + 2$

$\Rightarrow$ $x + 2y + 4 = 0$

(C) *(d)* $\left(3, -\frac{7}{2}\right)$

Explanation: Let point C divides AB in the ratio m_1 and m_2.



$$\text{Then, } (x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{1\times6 + 3\times2}{1+3}, \frac{1\times(-5) + 3(-3)}{1+3}\right)$$

$$= \left(\frac{12}{4}, \frac{-14}{4}\right) = \left(3, -\frac{7}{2}\right)$$

(D) *(c)* $\frac{\sqrt{5}}{2}$

Explanation: We have, A(2, –3) and $C\left(3, -\frac{7}{2}\right)$

$$AC = \sqrt{(3-2)^2 + \left(-\frac{7}{2}+3\right)^2}$$

$$= \sqrt{1^2 + \left(-\frac{1}{2}\right)^2}$$

$$= \sqrt{1+\frac{1}{4}} = \frac{\sqrt{5}}{2}$$

(E) (a) $\frac{3\sqrt{5}}{2}$

Explanation: We have, $C\left(3, -\frac{7}{2}\right)$ and B(6, –5)

$$CB = \sqrt{(6-3)^2 + \left(-5+\frac{7}{2}\right)^2}$$

$$= \sqrt{3^2 + \left(-\frac{3}{2}\right)^2}$$

$$= \sqrt{9+\frac{9}{4}} = \frac{\sqrt{45}}{4} = \frac{3\sqrt{5}}{2}$$

20. *The corner points of a square plot are (1, 2), (2, 3), (3, 1) (0, –4). Plot is located in an industrial area to build a well known company showroom.*

Based on the above information, answer the following questions:

(A) Find the distance between (1, 2) and (3, 1) and also find the slope of (1, 2) and (3, 1).

(B) Find the distance between (2, 3) and (0, –4) and also find the slope of (2, 3) and (0, –4).

(C) Determine ∠B of the triangle with vertices A(–2, 1), B(2, 3) and C(–2, –4).

Ans. (A) Here, A = (1, 2)

B = (3, 1)

$$AB = \sqrt{(3-1)^2 + (1-2)^2}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5}$$

Here, $x_1 = 2, x_2 = 0$

$y_1 = 3, y_2 = -4$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-4-3}{0-2}$$

$$= \frac{-7}{-2}$$

$$= \frac{7}{2}$$

(B) Here, $A = (2, 3)$

$B = (0, -4)$

So, $$AB = \sqrt{(0-2)^2 + (-4-3)^2}$$

$$= \sqrt{4+49}$$

$$= \sqrt{53}$$

Here, $x_1 = 1, x_2 = 3$

$y_1 = 2, y_2 = 1$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-2}{3-1}$$

$$= \frac{-1}{2}$$

(C) Slope of line, AB $= \frac{3-1}{2+2} = \frac{2}{4} = \frac{1}{2} = m_1$ (say)

Slope of line, BC $= \frac{-4-3}{-2-2} = \frac{7}{4} = m_2$

$$\tan B = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{7}{4} - \frac{1}{2}}{1 + \frac{1}{2}\cdot\frac{7}{4}}\right|$$

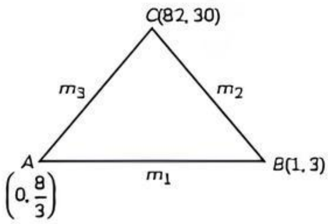
$$\angle B = \tan^{-1}\left(\frac{2}{3}\right)$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

21. Check whether the points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are the vertices of a triangle or not? **[Delhi Gov. QB 2022]**

Ans.



$$m_1 = \text{slope of AB} = \frac{3-\frac{8}{3}}{1-0} = \frac{1}{3}$$

$$m_2 = \text{slope of BC} = \frac{30-3}{82-1} = \frac{1}{3}$$

$\because \quad m_1 = m_2$

$\therefore$ A, B, C are collinear and will not form a triangle.

22. Find the point on y-axis whose perpendicular distance from the line $4x - 3y - 12 = 0$ is 3. **[Diksha]**

Ans. Let the required point be P(0, α). It is given that the length of the perpendicular from P(0, α) on $4x - 3y - 12 = 0$ is 3.

Now, perpendicular distance of a line $4x - 3y - 12 = 0$ from a point (0, α) is

$$D = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

$$\left|\frac{4 \times 0 - 3\alpha - 12}{\sqrt{4^2 + (-3)^2}}\right| = 3$$

$$\left|\frac{-3\alpha - 12}{\sqrt{16+9}}\right| = 3$$

$$\frac{|3\alpha + 12|}{5} = 3$$

$$|3\alpha + 12| = 15$$

$$|\alpha + 4| = 5$$

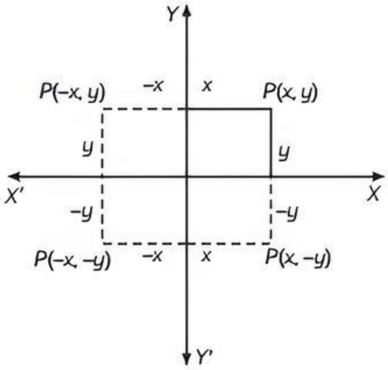
$$\alpha + 4 = \pm 5$$

$$\alpha = 1, -9$$

Hence, the points are (0, 1) and (0, –9).

23. If the sum of the distance of a moving point in a plane from the axes is 1, then find the locus of the point. **[NCERT Exemplar]**

Ans. Let the coordinates of a moving point P be (a, b).



Given that the sum of the distance from the axes to the point is always 1.

$\therefore \quad |x| + |y| = 1$

$\Rightarrow \quad \pm x \pm y = 1$

$\Rightarrow -x - y = 1, x + y = 1, -x + y = 1$ and $x - y = 1$

Hence, these equations gives us the locus of the point P which is a square.

24. If A(1, –1), B(2, 3) and C(–4, –2) are three points, find the angle between BA and BC.

Ans. Let m_1 and m_2 be the slopes of BA and BC respectively. Then,

$$m_1 = \frac{3+1}{2-(1)} = \frac{4}{1} = 4, \text{ and } m_2 = \frac{-2-3}{-4-2} = \frac{5}{6}$$

Let θ be the acute angle between BA and BC. Then,

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{5}{6} - 4}{1 + \frac{5}{6} \times 4}\right|$$

$$= \left|\frac{\frac{-19}{6}}{\frac{26}{6}}\right| = \left|\frac{-19}{26}\right|$$

$$\tan\theta = \frac{19}{26}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{19}{26}\right)$$

25. If a, b, c are in A.P. then find the points where the straight line $ax + by + c = 0$ will always pass. **[NCERT Exemplar]**

Ans. Given line is $ax + by + c = 0$ ...(i)

Since, a, b are c are in A.P., we get

$$b = \frac{a+c}{2}$$

Or, $a - 2b + c = 0$ ...(ii)

On comparing equation. (i) and (ii), we get

$x = 1$ and $y = -2$

So, (1, –2) lies on the line.

26. Find the coordinates of a point on $x + y + 3 = 0$ whose distance from $x + 2y + 2 = 0$ is $\sqrt{5}$. **[Diksha]**

Ans. Let the required point be (x_1, y_1).

Since, it lies on $x + y + 3 = 0$

$$x_1 + y_1 + 3 = 0 \quad ...(i)$$

Length of the perpendicular from (x_1, y_1) to

$x + 2y + 2 = 0$ is $\sqrt{5}$.

$$\left|\frac{x_1 + 2y_1 + 2}{\sqrt{1^2 + 2^2}}\right| = \sqrt{5}$$

$$x_1 + 2y_1 + 2 = \pm 5 \quad ...(ii)$$

or, $x_1 + 2y_1 - 3 = 0$ and $x_1 + 2y_1 + 7 = 0$

Solving eq. (i) and (ii), we get

$x_1 = -9, y_1 = 6$ and $x_1 = 1, y_1 = -4$

Hence, the points are (–9, 6) and (1, –4).

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

27. What is the value of y so that line through (3, y) and (2, 7) is parallel to the line through (–1, 4) and (0, 6)? **[Delhi Gov. QB 2022]**

Ans. Let m_1 be the slope of the line passing through (3, y) and (2, 7) and m_2 be the slope of the line passing through (–1, 4) and (0, 6).

$$\therefore \quad m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-y}{2-3}$$

$$= \frac{7-y}{-1} = y - 7$$

and $$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{0+1}$$

$$= \frac{6-4}{0+1} = \frac{2}{1} = 2$$

For both the lines to be parallel, we must have,

$$m_1 = m_2$$

$\Rightarrow \quad y - 7 = 2$

$\Rightarrow \quad y = 9$

Hence, the value of y is 9.

28. Find the obtuse angle between the lines $x - 2y + 3 = 0$ and $3x + y - 1 = 0$. **[Diksha]**

Ans. Let m_1 and m_2 be the slope of the straight lines $x - 2y + 3 = 0$ and $3x + y - 1 = 0$

$$m_1 = \frac{1}{2} \text{ and } m_2 = -3$$

Let θ be the angle between the given lines. Then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{1}{2}+3}{1-\left(\frac{1}{2}\right)(-3)}\right|$$

$$\tan\theta = \left|\frac{\frac{7}{2}}{-\frac{1}{2}}\right|$$

$\tan\theta = |-7|$

$\tan\theta = 7$

$\theta = \tan^{-1}(7)$

The acute angle between the lines is $\tan^{-1}(7)$ and the obtuse angle is $\pi - \tan^{-1}(7)$.

29. If the angle between two lines is $\frac{\pi}{4}$ and the slope of the line is $\frac{1}{2}$, find the slope of other line. **[Diksha]**

Ans. We know that acute angle θ between two lines with slope m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

Let, $m_1 = \frac{1}{2}$ and $m_2 = m$ be the slope of the other line.

It is given that $\theta = \frac{\pi}{4}$

Substituting $m_1 = \frac{1}{2}$, $m_2 = m$ and $\theta = \frac{\pi}{4}$, we obtain,

$$\tan\frac{\pi}{4} = \left|\frac{m - \frac{1}{2}}{1 + m \times \frac{1}{2}}\right|$$

$$1 = \left|\frac{2m-1}{2+m}\right|$$

$$\left|\frac{2m-1}{2+m}\right| = \pm 1$$

On solving further, we obtain

$$m = 3 \text{ or } m = -\frac{1}{3}$$

Hence, slope of the line is 3 or $-\frac{1}{3}$.

30. Show that the tangent of an angle between the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{a}-\frac{y}{b}=1$ is $\frac{2ab}{a^2-b^2}$.

[NCERT Exemplar]

Ans. Slope of the line $\frac{x}{a}+\frac{y}{b}=1$ is : $m_1 = -\frac{b}{a}$

Slope of the line $\frac{x}{a}-\frac{y}{b}=1$ is : $m_2 = \frac{b}{a}$

Let θ be the angle between the given lines. Then

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$= \left|\frac{-\frac{b}{a}-\frac{b}{a}}{1+\left(\frac{-b}{a}\right)\left(\frac{b}{a}\right)}\right|$$

$$= \left|\frac{\frac{-2b}{a}}{\frac{a^2-b^2}{a^2}}\right|$$

Thus., $\tan\theta = \frac{2ab}{a^2-b^2}$

31. Find the angle between the negative x-axis and the line joining the points (4, –2) and (5, –3).

Ans. Let m_1 be the slope of x-axis.

Then, $m_1 = 0$

Let m_2 be the slope of the line passing through the points (4, – 2)and (5, – 3).

Then, $m_2 = \frac{-3-(-2)}{5-4} = -1$

Let θ be the angle between the two lines.

Then, $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right| = \left|\frac{0-(-1)}{1+(0)(-1)}\right| = 1$

∴ $\theta = 45°$

Hence, the required angle is 45°.

32. In what direction should a line be drawn through the point (1, 2) so that its point of intersection with the line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point.

[NCERT Exemplar]

Ans. Let the slope of the line be m. Also, the line passes through the point A(1, 2).

∴ Equation of line is $y - 2 = m(x - 1)$

or $mx - y + 2 - m = 0$ ...(i)

Also the equation of the given line is

$x + y - 4 = 0$ (ii)

Let these lines meet at point B.

Solving (i) and (ii), we get

$$B = \left(\frac{m+2}{m+1}, \frac{3m+2}{m+1}\right)$$

Now, given that $AB = \frac{\sqrt{6}}{3}$

$\Rightarrow$ $AB^2 = \frac{6}{9}$

[On squaring both sides]

$$\Rightarrow \left(\frac{m+2}{m+1}-1\right)^2+\left(\frac{3m+2}{m+1}-2\right)^2 = \frac{6}{9}$$

$$\Rightarrow \left(\frac{1}{m+1}\right)^2+\left(\frac{m}{m+1}\right)^2 = \frac{2}{3}$$

$$\Rightarrow \frac{1+m^2}{(1+m)^2} = \frac{2}{3}$$

$\Rightarrow$ $3 + 3m^2 = 2 + 2m^2 + 4m$

$\Rightarrow$ $m^2 - 4m + 1 = 0$

∴ $m = \frac{4\pm\sqrt{16-4}}{2} = 2\pm\sqrt{3}$

$= 2+\sqrt{3}$ or $2-\sqrt{3}$

$\tan\theta = 2+\sqrt{3}$ or $2-\sqrt{3}$

$\theta = 75°$ or $\theta = 15°$.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

33. Find the angle between the lines $y = (2-\sqrt{3})(x+5)$ and $y = (2+\sqrt{3})(x-7)$

[NCERT Exemplar]

Ans. Given equations are $y = (2-\sqrt{3})(x+5)$ and $(2+\sqrt{3})(x-7)$.

The given equation can be written as

$\Rightarrow \quad y = (2-\sqrt{3})x + (2-\sqrt{3})5$ ...(i)

$\Rightarrow \quad y = (2+\sqrt{3})x - 7(2+\sqrt{3})$ ...(ii)

Now, we have to find the slope of equation (i).

Since, the equation (i) is in $y = mx + b$ form, we can easily see that the slope (m_1) is $(2-\sqrt{3})$.

Now, the slope (m_2) of equation (ii) is $(2+\sqrt{3})$.

Let θ be the angle between the given two lines.

$$\tan\theta = \left|\frac{m_1 - m_1}{1 + m_1 m_2}\right|$$

Putting the values of m_1 and m_2 in above equation, we get

$$\Rightarrow \quad \tan\theta = \left|\frac{2-\sqrt{3}-(2+\sqrt{3})}{1+(2-\sqrt{3})(2+\sqrt{3})}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{2-\sqrt{3}-2-\sqrt{3}}{1+[(2)^2-(\sqrt{3})^2]}\right|$$

$[\because (a-b)(a+b) = (a^2-b^2)]$

$$\Rightarrow \quad \tan\theta = \left|\frac{-2\sqrt{3}}{1+[4-3]}\right|$$

On simplifying, we get

$$\Rightarrow \quad \tan\theta = \left|\frac{-2\sqrt{3}}{1+1}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{-2\sqrt{3}}{2}\right|$$

$\Rightarrow \quad \tan\theta = \sqrt{3}$ or $-\sqrt{3}$

$\Rightarrow \quad \theta = \tan^{-1}(\sqrt{3})$ or $(-\sqrt{3})$

$\Rightarrow \quad \theta = 60°$ or $120°$

Hence, the required angle is 60° or 120°.

34. If the angle between two lines is $\frac{5\pi}{4}$ and the slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

Ans. Given that slope of one line is $m_1 = \frac{1}{2}$.

Let, m_2 be the slope of the other line.

Also, given that the angle between the two lines is $\theta = \frac{\pi}{4}$

Now, $$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \quad \tan\frac{\pi}{4} = \left|\frac{\frac{1}{2} - m_2}{1 + \left(\frac{1}{2}\right)m_2}\right|$$

$$\Rightarrow \quad 1 = \left|\frac{1-2m_2}{2+m_2}\right|$$

$$\Rightarrow \quad \pm 1 = \frac{1-2m_2}{2+m_2}$$

$$\therefore \quad \frac{1-2m_2}{2+m_2} = 1$$

$\Rightarrow \quad 1 - 2m_2 = 2 + m_2$

$\Rightarrow \quad m_2 = -\frac{1}{3}$

$$\therefore \quad \frac{1-2m_2}{2+m_2} = -1$$

$\Rightarrow \quad 1 - 2m_2 = -2 - m_2$

$\Rightarrow \quad m_2 = 3$

Hence, the slope of the other line is $-\frac{1}{3}$ or 3.

35. Find the points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$.

[NCERT Exemplar]

Ans. Let (x_1, y_1) be any point lying in the equation $x + y = 4$.

$\therefore \quad x_1 + y_1 = 4$ ...(i)

Distance of the point (x_1, y_1) from the equation $4x + 3y = 10$ is

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

$$\Rightarrow \quad 1 = \left|\frac{4x_1 + 3y_1 - 10}{\sqrt{(4)^2 + (3)^2}}\right|$$

On simplifying, we get

$$\Rightarrow \quad 1 = \left|\frac{4x_1+3y_1-10}{\sqrt{16+9}}\right|$$

$$\Rightarrow \quad 1 = \left|\frac{4x_1+3y_1-10}{5}\right|$$

$\Rightarrow$ $4x_1 + 3y_1 - 10 = \pm 5$

$4x_1 + 3y_1 - 10 = 5$ or $4x_1 + 3y_1 - 10 = -5$

$4x_1 + 3y_1 = 5 + 10$ or $4x_1 + 3y_1 = -5 + 10$

$4x_1 + 3y_1 = 15$ ...(ii)

Or $4x_1 + 3y_1 = 5$ ...(iii)

From equation (i), we have $y_1 = 4 - x_1$ ...(iv)

Putting the value of y_1 in equation (ii), we get

$4x_1 + 3(4 - x_1) = 15$

$\Rightarrow$ $4x_1 + 12 - 3x_1 = 15$

$\Rightarrow$ $x_1 = 15 - 12$

$\Rightarrow$ $x_1 = 3$

Putting the value of x_1 in equation (iv), we get

$y_1 = 4 - 3$

$\Rightarrow$ $y_1 = 1$

Putting the value of $y_1 = 4 - x_1$ in equation (iii), we get

$4x_1 + 3(4 - x_1) = 5$

$\Rightarrow$ $4x_1 + 12 - 3x_1 = 5$

$\Rightarrow$ $x_1 = 5 - 12$

$\Rightarrow$ $x_1 = -7$

Putting the value of x_1 in equation (iv), we get

$y_1 = 4 - (-7)$

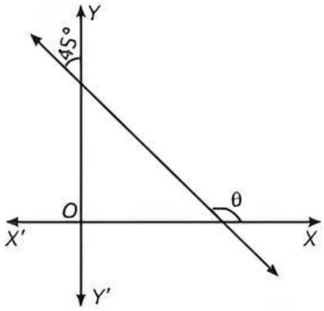
$\Rightarrow$ $y_1 = 4 + 7$

$\Rightarrow$ $y_1 = 11$

Hence, the required points on the given line are (3, 1) and (-7, 11).

36. Find the slope of the line, which makes an angle of 45° with the positive direction of y-axis measured anticlockwise.

Ans. Given that a line makes an angle of 45° with the positive direction of y - axis measured anticlockwise.



So, the line makes an angle of 135° with the positive direction of x-axis measured anticlockwise.

$\therefore$ $\theta = 135°$

Hence, slope of line, $m = \tan\theta$

$= \tan 135°$

$= \tan(90° + 45°)$

$= -\cot 45°$

$= -1$

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

37. Find the points on the x-axis, whose distance from the line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units.

Ans. Given the equation of the line is

$$\frac{x}{3}+\frac{y}{4}=1$$

$\Rightarrow$ $4x + 3y - 12 = 0$

which is of the form $Ax + By + C = 0$ with $A = 4, B = 3, C = -12$.

Let $(a, 0)$ be the required point on x-axis.

Let, $x_1 = a, y_1 = 0$

Now, the distance of a point (x_1, y_1) from the given line is

$$d = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

$$= \frac{|(4)(a)+(3)(0)-12|}{\sqrt{(4)^2+(3)^2}}$$

$$= \frac{|4a-12|}{5}$$

Since, the point $(a, 0)$ is at a distance of 4 units from the given line.

$\therefore$ $d = 4$

$$\Rightarrow \quad \frac{|4a-12|}{5} = 4$$

$\Rightarrow$ $|4a - 12| = 20$

$\Rightarrow$ $4a - 12 = \pm 20$

$\Rightarrow$ $a = 8, -2$

Hence, the required points are (8, 0) and (– 2, 0).

38. A variable line passes through a fixed point P. The algebraic sum of the perpendiculars drawn from the points (2, 0), (0, 2) and (1, 1) on the line is zero. Find the coordinates of the point P. [NCERT Exemplar]

Ans. Let the variable line be $ax + by = 1$

We know that, length of the perpendicular from (p, q) to the line $ax + by + c = 0$ is

$$d = \left|\frac{ap+bq+c}{\sqrt{a^2+b^2}}\right|$$

Now, perpendicular distance from A(2,0) is

$$d_1 = \left|\frac{2\times a + 0\times b - 1}{\sqrt{a^2+b^2}}\right|$$

$$= \frac{2a-1}{\sqrt{a^2+b^2}}$$

Now, perpendicular distance from B(0,2) is

$$d_2 = \left|\frac{0\times a + 2\times b - 1}{\sqrt{a^2+b^2}}\right|$$

$$= \frac{2b-1}{\sqrt{a^2+b^2}}$$

Now, perpendicular distance from C(1, 1) is

$$d_3 = \left|\frac{1\times a + 1\times b - 1}{\sqrt{a^2+b^2}}\right|$$

$$= \frac{a+b-1}{\sqrt{a^2+b^2}}$$

It is given that the algebraic sum of the perpendicular from the given points (2, 0), (0, 2) and (1, 1) to this line is zero.

$$d_1 + d_2 + d_3 = 0$$

Substituting the values, we get

$$\therefore \quad \frac{2a-1}{\sqrt{a^2+b^2}} + \frac{2b-1}{\sqrt{a^2+b^2}} + \frac{a+b-1}{\sqrt{a^2+b^2}} = 0$$

$\Rightarrow \quad 2a - 1 + 2b - 1 + a + b - 1 = 0$

$\Rightarrow \quad 3a + 3b - 3 = 0$

$\Rightarrow \quad a + b - 1 = 0$

$\Rightarrow \quad a + b = 1$

So, the equation $ax + by = 1$ represents a family of straight lines passing through a fixed point.

Comparing the equation $ax + by = 1$ and $a + b = 1$, we get

$x = 1$ and $y = 1$

So, the coordinates of fixed points is (1, 1).

39. P_1, P_2 are points on either of the two lines $y - \sqrt{3}|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from P_1, P_2 on the bisector of the angle between the given lines. [NCERT Exemplar]

Ans. Given lines are $y - \sqrt{3}|x| = 2$. If $x \geq 0$, then

$$y - \sqrt{3}x = 2 \quad \text{...(i)}$$

If $x < 0$, then

$$y + \sqrt{3}x = 2 \quad \text{...(ii)}$$

On adding equation (i) and (ii), we get

$$y - \sqrt{3}x + y + \sqrt{3}x = 2 + 2$$

$\Rightarrow \quad 2y = 4$

$\Rightarrow \quad y = 2$

Substituting the value of $y = 2$ in equation (ii), we get

$$2 + \sqrt{3}x = 2$$

$\Rightarrow \quad \sqrt{3}x = 2 - 2$

$\Rightarrow \quad x = 0$

$\therefore$ Point of intersection of given lines is (0, 2).

Now, we find the slopes of given lines.

Slope of equation (i) is

$$y = \sqrt{3}x + 2$$

Comparing the above equation with $y = mx + b$, we get

$$m = \sqrt{3}$$

And we know that, $m = \tan\theta$

$\therefore \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \theta = 60° \quad [\because \tan 60° = \sqrt{3}]$

Slope of equation (ii) is

$$y = -\sqrt{3}x + 2$$

Comparing the above equation with

$y = mx + b$, we get

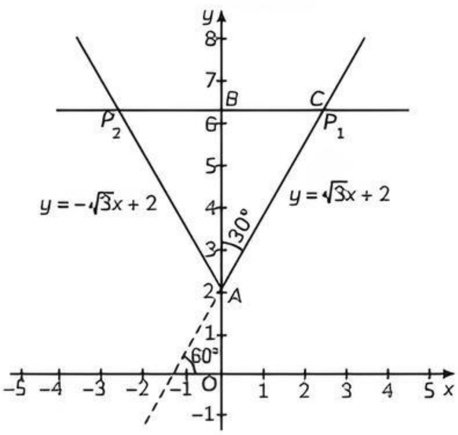
$$m = -\sqrt{3}$$

And we know that, $m = \tan\theta$

$\therefore \quad \tan\theta = -\sqrt{3}$

$\Rightarrow \quad \theta = (180° - 60°)$

$\Rightarrow \quad \theta = 120°$



In ΔACB,

$$\cos 30° = \frac{BA}{AC}$$

Given $\quad AC = 5$ units

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{BA}{5}$$

$$\Rightarrow \quad BA = \frac{5\sqrt{3}}{2}$$

$$OB = OA + AB$$

$$= 2+\frac{5\sqrt{3}}{2}$$

Hence, the coordinates of the foot of perpendicular is $\left(0, 2+\frac{5\sqrt{3}}{2}\right)$.

40. In the triangle ABC with vertices A(2, 3), B(4, −1), and C(1, 2), find the equation and length of altitude from vertex A.

Ans. Given, a triangle ABC with vertices A(2, 3), B(4, −1), and C(1, 2).

Let AD be the perpendicular form the vertex A.

Now, slope of BC i.e.

$$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{2-(-1)}{1-4} = \frac{3}{-3} = -1$$

Let m_2 be the slope of AD.

Since, AD is perpendicular to BC.

$$m_1 m_2 = -1$$

$$-1 m_2 = -1$$

$$\Rightarrow \quad m_2 = 1$$

So, slope of AD is 1.

Equation of line having one coordinate and slope is given by

$$y - y_1 = m(x - x_1)$$

So, equation of *AD* is

$$y - 3 = 1(x - 2)$$

$$\Rightarrow \quad y = x + 1$$

$$\Rightarrow \quad -x + y - 1 = 0$$

We know that the perpendicular distance of line $Ax + By + C = 0$ from point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Now, length of AD from vertex A(2, 3) is

$$\Rightarrow \quad \left|\frac{-3+2-1}{\sqrt{(-1)^2+1^2}}\right| = \left|\frac{-2}{\sqrt{2}}\right|$$

$$= \sqrt{2}$$

Hence, length of altitude AD is $\sqrt{2}$.

EQUATIONS OF LINE 2

TOPIC 1

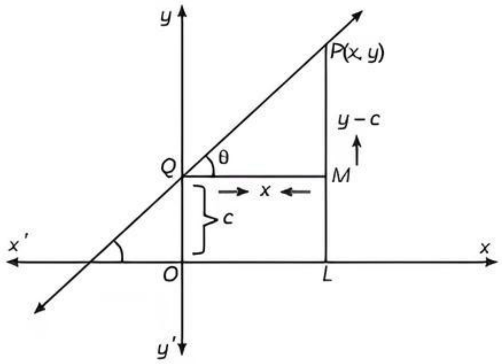
VARIOUS FORMS OF EQUATIONS OF A LINE

Slope Intercept Form of a Line

The equation of a line with slope m and making an intercept c on y-axis is given by $y = mx + c$.

Let the given line intersects y-axis at Q and makes an angle θ with x-axis. Then $m = \tan\theta$.

Let P(x, y) be any point on the line. Draw PL perpendicular to x-axis and QM $\perp$ PM.



Clearly, $\angle MQP = \theta$, $QM = OL = x$

And, $PM = PL - ML = PL - OQ = y - c$

From triangle, PMQ, we have

$$\tan\theta = \frac{PM}{QM} = \frac{y-c}{x}$$

$$\Rightarrow \quad m = \frac{y-c}{x}$$

$$\Rightarrow \quad y = mx + c$$

which is the required equation of the line.

Two-Point Form of a Line

The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Let m be the slope of the line passing through (x_1, y_1) and (x_2, y_2). Then,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

So, the equation of the line is

$$y - y_1 = m(x - x_1)$$

Substituting the value of m, we obtain

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

This is the required equation of the line in two-point form.

Point-Slope Form of a Line

The equation of a line which passes through the point (x_1, y_1) and has the slope 'm' is $y - y_1 = m(x - x_1)$.

Let the line pass through the point Q(x_1, y_1) and let P(x, y) be any point on the line.

Then, slope of the line is $= \dfrac{y - y_1}{x - x_1}$

But, the slope of the line is m.

$$\therefore \quad m = \frac{y - y_1}{x - x_1}$$

$$\Rightarrow \quad y - y_1 = m(x - x_1)$$

Hence, $y - y_1 = m(x - x_1)$ is the required equation of the line.

Example 2.1: **Find the equation of the line which passes through $(2, 2\sqrt{3})$ and inclined with the x-axis at an angle of 75°.** **[NCERT]**

Ans. We know that

Equation of line passing through point (x_0, y_0) with slope m is

$$y - y_0 = m(x - x_0)$$

Here, point $(x_0, y_0) = (2, 2\sqrt{3})$

Here, $x_0 = 2$ and $y_0 = 2\sqrt{3}$

Now, slope $= m = \tan\theta$

Given $\theta = 75°$

$$\therefore \quad m = \tan(75°)$$

$$= \tan(45 + 30)°$$

$$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$

$$\left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\therefore \qquad m = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

On putting value of m in $(y - y_0) = m(x - x_0)$, we get

$$(y-2\sqrt{3}) = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x-2)$$

$$(y-2\sqrt{3})(\sqrt{3}-1) = (\sqrt{3}+1)(x-2)$$

$$y(\sqrt{3}-1)-2\sqrt{3}(\sqrt{3}-1)$$

$$= x(\sqrt{3}+1)-2(\sqrt{3}+1)$$

$$y(\sqrt{3}-1)-2\sqrt{3}\times\sqrt{3}+2\sqrt{3}$$

$$= x(\sqrt{3}+1)-2\sqrt{3}-2$$

$$y(\sqrt{3}-1)-6+2\sqrt{3} = x(\sqrt{3}+1)-2\sqrt{3}-2$$

$$y(\sqrt{3}-1) = x(\sqrt{3}+1)-2\sqrt{3}-2+6-2\sqrt{3}$$

$$y(\sqrt{3}-1) = x(\sqrt{3}+1)-4\sqrt{3}+4$$

$$y(\sqrt{3}-1)-x(\sqrt{3}+1) = -4\sqrt{3}+4$$

$$x(\sqrt{3}+1)-y(\sqrt{3}+1) = 4\sqrt{3}-4$$

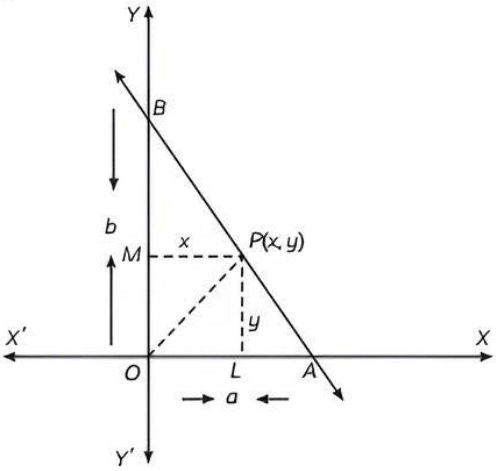
$$x(\sqrt{3}+1)-y(\sqrt{3}-1) = 4(\sqrt{3}-1)$$

The Intercept form of a Line

The equation of a line which cuts off intercepts a and b respectively from the x and y-axis is

$$\frac{x}{a}+\frac{y}{b}=1.$$

Let AB be the line which cuts off intercepts OA = a and OB = b on the x and y-axes respectively. Let P(x, y) be any point on the line. Draw PL ⊥ OX. Then, OL = x and PL = y



Clearly,

Area of ΔOAB = Area of ΔOPB

$$\Rightarrow \quad \frac{1}{2}OA.OB = \frac{1}{2}OA.PL + \frac{1}{2}PM.OB$$

$$\Rightarrow \quad \frac{1}{2}ab = \frac{1}{2}ay + \frac{1}{2}bx$$

$$\Rightarrow \quad ab = ay + bx$$

$$\Rightarrow \quad \frac{x}{a}+\frac{y}{b}=1$$

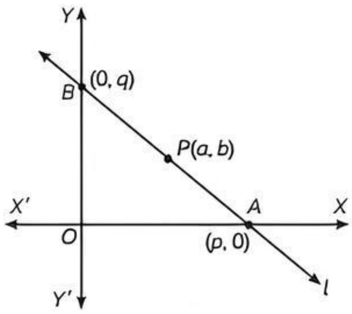
This is the equation of the line in the intercept form.

Example 2.2: Reduce the equation $3y + 1 = 0$ into intercepts form and find its intercepts on the axes. **[NCERT]**

Ans. Given the equation of the line is

$$3y + 1 = 0$$

$$y = -\frac{1}{3}$$



which is the required intercept form.

The given line has y-intercept as $-\frac{1}{3}$ but it has no x-intercept.

Example 2.3: P(a, b) is the midpoint of a line segment between axes. Show that equation of the line is $\frac{x}{a}+\frac{y}{b}=2$. **[NCERT]**

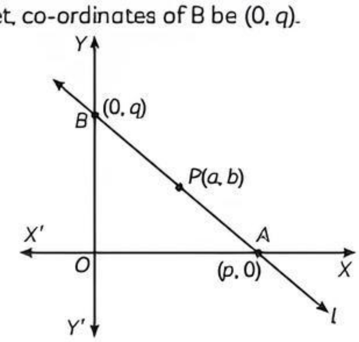
Ans. Plotting x-axis and y-axis

Let l be a line intersecting x-axis at A and y-axis at 0.

Let, P(a, b) be midpoint of AB.

Let, co-ordinates of A be (p, 0).

Let, co-ordinates of B be (0, q).



We know that mid-point of a line joining points

$$(x_1, y_1) \text{ and } (x_2, y_2) \text{ is } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Mid-point of a line joining points A(p, 0) and B(0, q) is P(a, b).

Putting values

$$(a, b) = \left(\frac{p+0}{2}, \frac{0+q}{2}\right)$$

$$(a, b) = \left(\frac{p}{2}, \frac{q}{2}\right)$$

$$a = \frac{p}{2}$$

$\Rightarrow$ $p = 2a$

$$b = \frac{q}{2}$$

$$2b = q$$

$$q = 2b$$

Or, $p = 2a$ and $q = 2b$

So, points are

$$A = (p, 0) = (2a, 0)$$

$$B = (0, q) = (0, 2b)$$

Finding equation of line by two point equation of line

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

For equation of line l passing through ($2a$, 0) and (0, $2b$).

Here, $x_1 = 2a, y_1 = 0$

$x_2 = 0, y_2 = 2b$

On putting values, we get

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$(y - 0) = \frac{2b - 0}{0 - 2a}(x - 2a)$$

$$(y - 0) = \frac{2b}{-2a}(x - 2a)$$

$$(y - 0) = \frac{-b}{a}(x - 2a)$$

$$ay = -bx + 2ab$$

$$ay + bx = 2ab$$

Dividing by ab on both sides, we get

$$\frac{ay}{ab} + \frac{bx}{ab} = \frac{2ab}{ab}$$

$$\frac{y}{b} + \frac{x}{a} = 2$$

$$\frac{x}{a} + \frac{y}{b} = 2$$

So, the equation of line is $\frac{x}{a} + \frac{y}{b} = 2$

Hence, proved.

Example 2.4: Find the distance of the line $4x + 7y + 5 = 0$ from the point (1, 2) along the line $2x - y = 0$.

Ans. There are two lines

Line CD: $2x - y = 0$ ...(i)

Line AB: $4x + 7y + 5 = 0$ ...(ii)

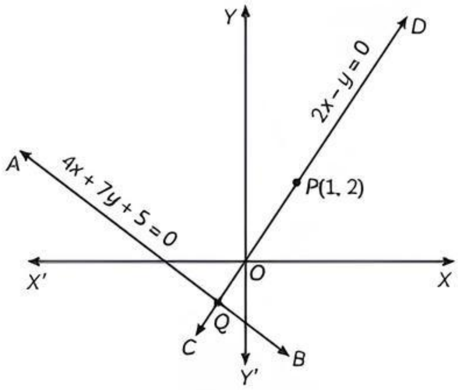
Both lines meet at Q.

Point P(1, 2) is on line CD.

We need to find distance PQ, in PQ, P is (1, 2)

We need to find point Q.

To find PQ, we must find coordinates of point Q



Point Q is the intersection of lines AB and CD.

From (i)

$$2x - y = 0$$

$$2x = y$$

$$y = 2x$$

Putting value of y in eq. (ii)

$$4x + 7(2x) + 5 = 0$$

$$4x + 14x + 5 = 0$$

$$18x = -5$$

$$x = \frac{-5}{18}$$

On putting, $x = \frac{-5}{18}$ in eq. (i), we get

$$2x - y = 0$$

$$2\left(\frac{-5}{18}\right) - y = 0$$

$$\frac{-5}{9} - y = 0$$

$$\frac{-5}{9} = y$$

$$y = \frac{-5}{9}$$

Hence, coordinates of point Q is $\left(\frac{-5}{18}, \frac{-5}{9}\right)$

Now, we have to find distance PQ.

We know that distance of points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance PQ where P(1, 2) and Q $\left(\frac{-5}{18}, \frac{-5}{9}\right)$ is

$$PQ = \sqrt{\left(\frac{-5}{18} - 1\right)^2 + \left(\frac{-5}{9} - 2\right)^2}$$

$$= \sqrt{\left(\frac{-5-18}{18}\right)^2 + \left(\frac{-5-18}{9}\right)^2}$$

$$= \sqrt{\left(\frac{-23}{18}\right)^2 + \left(\frac{-23}{9}\right)^2}$$

$$= \sqrt{\left(\frac{-23}{2 \times 9}\right)^2 + \left(\frac{-23}{9}\right)^2}$$

$$= \sqrt{\left(\frac{-23}{9}\right)^2 \cdot \left(\frac{1}{4} + 1\right)}$$

$$= \sqrt{\left(\frac{-23}{9}\right)^2 \cdot \left(\frac{1+4}{4}\right)}$$

$$= \sqrt{\left(\frac{-23}{9}\right)^2 \cdot \left(\frac{5}{4}\right)}$$

$$= \frac{-23}{9}\sqrt{\frac{5}{4}}$$

$$= \frac{-23}{9}\sqrt{\frac{5}{2^2}}$$

$$= \frac{-23}{9} \frac{\sqrt{5}}{2}$$

$$= \frac{-23\sqrt{5}}{18}$$

But distance is always positive

Hence, distance of PQ = $\frac{23\sqrt{5}}{18}$.

Example 2.5: Find the distance between parallel lines $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$. [NCERT]

Ans. The equation of the first line is

$$15x + 8y - 34 = 0,$$

which is of the form $Ax + By + C_1 = 0$

with $A = 15$, $B = 8$, $C_1 = -34$.

The equation of the second line is

$$15x + 8y + 31 = 0$$

which is of the form $Ax + By + C_2 = 0$

with $A = 15$, $B = 8$, $C_2 = 31$.

Now, the distance between the given parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|-34 - 31|}{\sqrt{(15)^2 + (8)^2}} = \frac{65}{17}$$

Hence, the required distance is $\frac{65}{17}$ units.

Example 2.6: Find the equation of a line parallel to y-axis and passing through (–2, 5). [NCERT]

Ans. The equation of a line parallel to y-axis is of the form $x = a$. ...(i)

Given that the line passes through the point (–2, 5).

Putting $x = -2$ and $y = 5$ in (i), we get

$$-2 = a$$

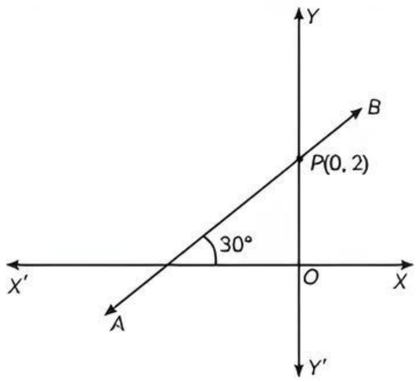
Putting the value of a in (i), we get

$$x = -2.$$

which is the required equation.

Example 2.7: Find the equation of the line which intersects the y-axis at a distance of 2 units above the origin and makes an angle of 30° with the positive direction of the x-axis. [NCERT]

Ans. Line AB intersects the y-axis 2 units above origin.



At y-axis, x will always be 0.

$\therefore$ Line AB cuts y – axis at P(0, 2)

Also, line AB makes an angle 30° with the x-axis.

$\therefore$ Slope = $\tan\theta = m$

$$= \frac{1}{\sqrt{3}}$$

We know that equation of line passing through (x_0, y_0) and having slope m is

$$(y - y_0) = m(x - x_0)$$

Putting, $x_0 = 0$, $y_0 = 2$ and $m = \frac{1}{\sqrt{3}}$

We have

$$(y-2) = \frac{1}{\sqrt{3}}(x-0)$$

$$(y-2) = \frac{1}{\sqrt{3}}x$$

$$\sqrt{3}(y-2) = x$$

$$\sqrt{3}y - 2\sqrt{3} = x$$

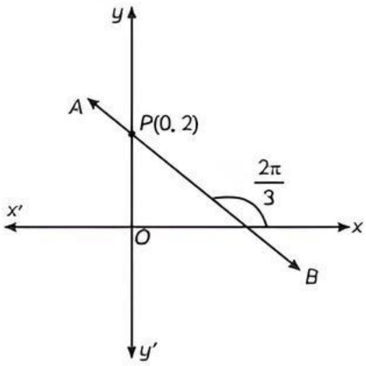
$$\sqrt{3}y - x - 2\sqrt{3} = 0$$

Hence, the required equation is

$$\sqrt{3}y - x - 2\sqrt{3} = 0$$

Example 2.8: **Find equation of the line through the point(0, 2) making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of line parallel to it and crossing the y-axis at a distance of 2 units below the origin.** **[NCERT]**

Ans. Let AB be the line passing through P(0, 2) and making an angle $\frac{2\pi}{3}$ with positive x-axis.



$$\text{Slope of line AB} = \tan\theta = m$$

$$= \tan\left(\frac{2\pi}{3}\right)$$

$$= \tan(120°)$$

$$= \tan(180° - 60°)$$

$$= -\tan(60°)$$

$$= -\sqrt{3} \quad (\tan 60° = \sqrt{3})$$

We know that

Equation of line passing through (x_0, y_0) and having slope m

$$(y - y_0) = m(x - x_0)$$

Equation of line AB passing through (0, 2) and having slope $m = -\sqrt{3}$.

$$(y-2) = -\sqrt{3}(x-0)$$

$$(y-2) = -\sqrt{3}x$$

$$y + \sqrt{3}x = 2$$

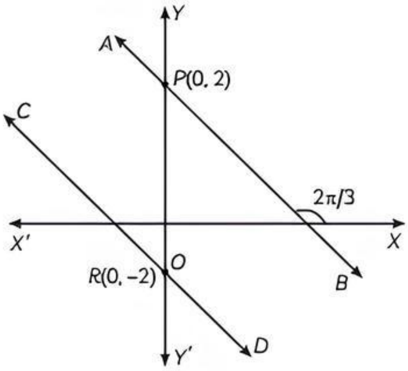
$$\sqrt{3}x + y = 2$$

$$\sqrt{3}x + y - 2 = 0$$

Hence, equation of line AB is $\sqrt{3}x + y - 2 = 0$.

Also,

We have to find equation of line which is parallel to line AB and crossing at a distance of 2 unit below the origin.



Let CD be the line parallel to AB and passing through point R(0, – 2).

We know that if two lines are parallel, then their slopes are equal

Therefore,

Slope of CD = Slope of AB

Slope of CD = $-\sqrt{3}$

Now,

Equation of line passing through point (x_0, y_0) and having slope m

$$(y - y_0) = m(x - x_0)$$

Equation of line CD passing through (0, –2) and slope $m = -\sqrt{3}$.

$$(y-(-2)) = -\sqrt{3}(x-0)$$

$$(y+2) = -\sqrt{3}(x)$$

$$y + \sqrt{3}x + 2 = 0$$

$$\sqrt{3}x + y + 2 = 0$$

Hence, equation of line CD = $\sqrt{3}x + y + 2 = 0$

Example 2.9: **The owner of a milk store finds that he can sell 980 litres of milk each week at ₹ 14/litre and 1220 litres of milk each week at ₹ 16/Litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at 17/litre?** **[NCERT]**

Ans. Let selling price be P along x-axis and demand of milk be D along y-axis.

We know that the equation of line is

$$y = mx + c$$

Here, P is along x-axis and D is along y-axis.

So, our equation becomes

$$D = mP + c \quad \text{...(i)}$$

Now,

Owner sells 980 litre milk at ₹ 14/litre.

So, D = 980 and P = 14 satisfies the equation.

Putting values in (i), we get

$$980 = 14m + c \quad \text{...(ii)}$$

Owner sells, 1220 litre milk at ₹ 16/Litre.

So, D =1220 and P =16 satisfies the equation.

Putting values in (i), we get

$$1220 = 16m + c \quad \text{...(iii)}$$

So, our equations are

$$980 = 14m + c$$
$$1220 = 16m + c$$

From (ii)

$$980 = 14m + c$$
$$980 - 14m = c$$

Putting value of c in (iii)

$$1220 = 16m + 980 - 14m$$
$$1220 - 980 = 16m - 14m$$
$$240 = 2m$$
$$\frac{240}{2} = m$$
$$120 = m$$
$$m = 120$$

Putting, $m = 120$ in (ii), we get

$$980 = 14m + c$$
$$980 - 14m = c$$
$$980 - 14(120) = c$$
$$980 - 1680 = c$$
$$-700 = c$$
$$c = -700$$

Putting value of m and c in (i)

$$D = mP + c$$
$$D = 120P - 700$$

Hence, the required equation is $D = 120P - 700$

We need to find how many litres could he sell weekly at ₹ 17/litres.

i.e. we need to find D when P = 17

On putting P = 17 in the equation, we get

$$D = 120P - 700$$
$$D = 120(17) - 700$$
$$D = 2040 - 700$$
$$D = 1340$$

Hence, when price is ₹ 17/litre, 1340 litres of milk could be sold.

Example 2.10: Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$. [NCERT]

Ans. Given equation of lines,

$$\sqrt{3}x + y = 1 \quad \text{...(i)}$$
$$x + \sqrt{3}y = 1 \quad \text{...(ii)}$$

We know that,

Angle θ between two lines can be found by using formula

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

Let slope of line (i) be m_1 and slope of line (ii) be m_2

Calculating m_1

From (i), $\sqrt{3}x + y = 1$

$$y = -\sqrt{3}x + 1$$

From above, the equation is of the form

$$y = mx + c$$

Where m is the slope

Thus, $m_1 = -\sqrt{3}$

Calculating m_2

From (i)

$$x + \sqrt{3}y = 1$$
$$\sqrt{3}y = -x + 1$$
$$y = \frac{-x + 1}{\sqrt{3}}$$
$$y = \left(\frac{-1}{\sqrt{3}}\right)x + \left(\frac{1}{\sqrt{3}}\right)$$

From above, the equation is of the form

$$y = mx + c$$

Where m is the slope

Thus, $m_2 = \frac{-1}{\sqrt{3}}$

Now,

Angle between lines $\sqrt{3}x + y = 1$ *and* $x + \sqrt{3}y = 1$ is

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Putting values

$$\tan\theta = \left|\frac{-\sqrt{3} - \left(\frac{-1}{\sqrt{3}}\right)}{1 + (-\sqrt{3})\left(\frac{-1}{\sqrt{3}}\right)}\right|$$

$$\tan\theta = \left|\frac{-\sqrt{3}+\frac{1}{\sqrt{3}}}{1+1}\right|$$

$$\tan\theta = \left|\frac{\frac{(-\sqrt{3})(\sqrt{3})+1}{\sqrt{3}}}{2}\right|$$

$$\tan\theta = \left|\frac{-3+1}{2\sqrt{3}}\right|$$

$$\tan\theta = \left|\frac{-1}{\sqrt{3}}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

We know that, $\tan 30° = \frac{1}{\sqrt{3}}$

$\tan\theta° = \tan 30°$

Hence, $\theta = 30°$

Thus, the acute angle between the lines (i) and (ii) is 30°

And obtuse angle (ϕ) between these two lines is

$\phi = 180° - \theta$

$\phi = 180° - 30°$

$\phi = 150°$

Thus, the required angle between the line is 30°or 150°.

Example 2.11: **Find the values of k for which the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ is:**

(A) Parallel to the x-axis

(B) Parallel to the y-axis

(C) Passing through origin **[NCERT]**

Ans. (A) Any line parallel to x-axis is of the form

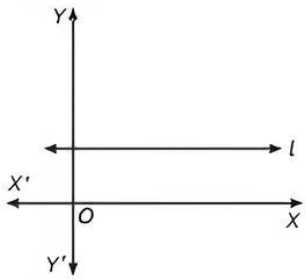
$y = p$

Where p is constant.

So, there is no term of x.

Since,

$(k-3)x-(4-k^2)y+k^2-7k+6=0$



Parallel to x-axis

Hence,

$(k-3)x = 0$

$k-3 = \frac{0}{x}$

$\Rightarrow k-3 = 0$

$k = 3$

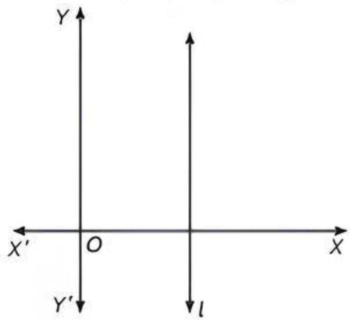
(B) Any line parallel to y-axis is of the form

$x = q$

Where q is constant

So, there is no term of y.

Since, line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ is



Parallel to y-axis

Hence,

$-(4-k^2)y = 0$

$-(4-k^2) = \frac{0}{y}$

$-4+k^2 = 0$

$k^2 = 4$

$k = \sqrt{4}$

$k = \pm 2$

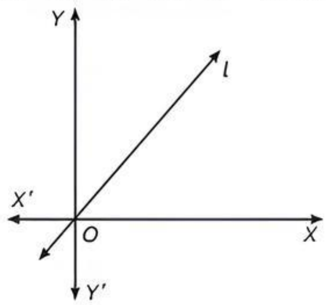
Hence, $k = 2$ or -2

(C) If the line passing through the origin i.e. (0, 0) will satisfy the equation of line

Putting $x = 0$ and $y = 0$ in equation

$(k-3)x-(4-k^2)y+k^2-7k+6=0$

$(k-3)0-(4-k^2)0+k^2-7k+6=0$



$k^2-7k+6=0$

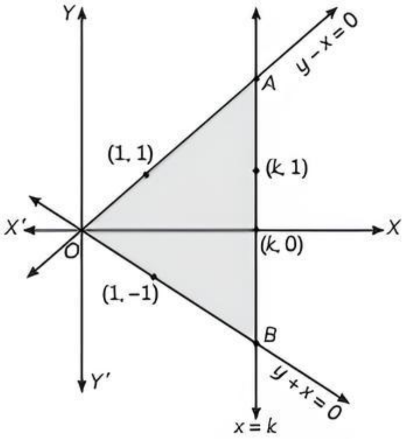
$k^2-6k-k+6=0$

$k(k-6)-1(k-6)=0$

$(k-1)(k-6)=0$

So, $k = 1$ or $k = 6$

Example 2.12: **Find the area of the triangle formed by the lines $y-x=0$, $x+y=0$ and $x-k=0$.**



[NCERT]

Ans. There are three lines given in the graph

$y - x = 0$ ...(i)

(0, 0) and (1, 1) satisfy this line

$x + y = 0$ ...(ii)

(0, 0) and (1, – 1) satisfy this line

$x - k = 0$ ...(iii)

(k, 0) satisfy this line.

Let, line (i) and (iii) intersect be point A and whereas line (ii) and (iii) intersect be point B

We need to find area triangle ΔOAB

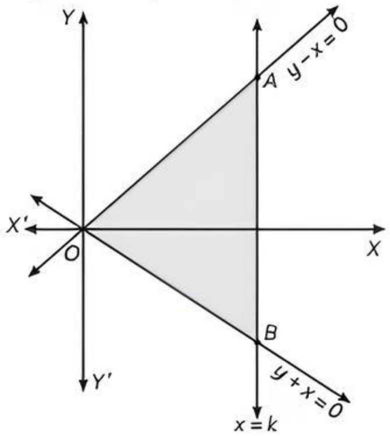
We know O(0, 0)

We need to find coordinates of A and B

Coordinate of point A

Point A is the intersection of $x = k$ and $y - x = 0$

Putting $x = k$ in equation (i)



$y - k = 0$

$y = k$

So, point A(k, k)

Coordinate of point B

Point B is the intersection of $x = k$ and $y + x = 0$

Putting $x = k$ in equation (2)

$y + k = 0$

$y = -k$

So, point B(k, – k).

We know that

Area of triangle whose vertices are

(x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

For ΔAOB,

$(x_1, y_1) = O(0, 0)$

$(x_2, y_2) = A(k, k)$

$(x_3, y_3) = B(k, -k)$

Area of triangle ΔAOB whose vertices are (0, 0) (k, k) and (k,– k)

$$= \frac{1}{2}|0(k-(-k)) + k(-k-0) + k(0-k)|$$

$$= \frac{1}{2}|0 + k(-k) + k(-k)|$$

$$= \frac{1}{2}|-k^2 - k^2|$$

$$= \frac{1}{2}|-2k^2|$$

$$= \frac{2k^2}{2}$$

$= k^2$ square units

Hence, required area of triangle is k^2 square units.

Example 2.13: If the lines $y = 3x + 1$ and $2y = x + 3$ are equally inclined to the line $y = mx + 4$, find the value of m. [NCERT]

Ans. The equations of the given lines are

$y = 3x + 1$ ...(i)

$2y = x + 3$ ...(ii)

$y = mx + 4$ ...(iii)

Slope of line (i), $m_1 = 3$

Slope of line (ii), $m_2 = \frac{1}{2}$

Slope of line (iii), $m_3 = m$

It is given that lines (i) and (ii) are equally inclined to line (iii), this means that the angle between lines (i) and (iii) equals the angle between lines (ii) and (iii)

$$\therefore \quad \left|\frac{m_1 - m_2}{1 + m_1 m_3}\right| = \left|\frac{m_2 - m_3}{1 + m_2 m_3}\right|$$

$$\Rightarrow \quad \left|\frac{3 - m}{1 + 3m}\right| = \left|\frac{\frac{1}{2} - m}{1 + \frac{1}{2}m}\right|$$

$$\left|\frac{3 - m}{1 + 3m}\right| = \left|\frac{1 - 2m}{m + 2}\right|$$

$$\Rightarrow \quad \frac{3-m}{1+3m} = \pm\left\{\frac{1-2m}{m+2}\right\}$$

$$\Rightarrow \quad \frac{3-m}{1+3m} = \frac{1-2m}{m+2}$$

or $$\frac{3-m}{1+3m} = -\left\{\frac{1-2m}{m+2}\right\}$$

If $\frac{3-m}{1+3m} = \frac{1-2m}{m+2}$ then

$$(3-m)(m+2) = (1-2m)(1+3m)$$

$$\Rightarrow m^2 + m + 6 = 1 + m - 6m^2$$

$$\Rightarrow 5m^2 + 5 = 0$$

$$\Rightarrow (m^2 + 1) = 0$$

$\Rightarrow m = \sqrt{-1}$, which is not real.

Hence, this case is not possible.

If $\frac{3-m}{1+3m} = \frac{1-2m}{m+2}$ then

$$(3-m)(m+2) = (1-2m)(1+3m)$$

$$\Rightarrow -m^2 + m + 6 = -(1 + m - 6m^2)$$

$$\Rightarrow 7m^2 - 2m - 7 = 0$$

$$\Rightarrow \quad m = \frac{2 \pm \sqrt{4-4(7)(-7)}}{2(7)}$$

$$\Rightarrow \quad m = \frac{2 \pm 2\sqrt{1+49}}{14}$$

$$\Rightarrow \quad m = \frac{1 \pm 5\sqrt{2}}{7}$$

Thus, the required values of m is

$$\Rightarrow \quad m = \frac{1 \pm 5\sqrt{2}}{7}.$$

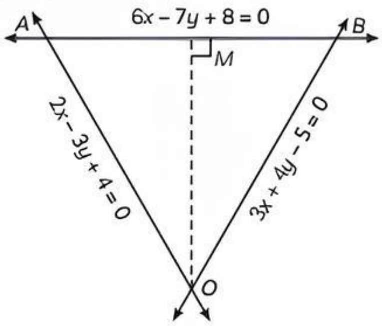
Example 2.14: A person standing at the junction (crossing) of two straight paths represented by the equations $2x - 3 + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find the equation of the path that he should follow. [NCERT]

Ans. Let equation of lines

OA : $2x - 3y + 4 = 0$ ...(i)

OB : $3x + 4y - 5 = 0$ ...(ii)

AB : $6x - 7y + 8 = 0$ ...(iii)



The two paths cross at point O.

∴ The person in standing at point O

From point O, if he has to reach line AB in the least time, least distance from point O to line AB will be perpendicular distance.

Let, OM ⊥ AB

We need to find equation of line OM.

First we find coordinates of point O.

Point O is the intersection of OA and OB.

From (i)

$$2x - 3y + 4 = 0$$

$$2x = 3y - 4$$

$$x = \frac{3y-4}{2}$$

$$x = \frac{3}{2}y - \frac{4}{2}$$

$$x = \frac{3}{2}y - 2$$

Putting value of x in (ii)

$$3x + 4y - 5 = 0$$

$$3\left(\frac{3}{2}y - 2\right) + 4y - 5 = 0$$

$$\frac{9}{2}y - 6 + 4y - 5 = 0$$

$$\frac{9}{2}y + 4y - 11 = 0$$

$$\frac{9y + 8y}{2} = 11$$

$$9y + 8y = 22$$

$$17y = 22$$

$$y = \frac{22}{17}$$

Putting value of y in (i)

$$2x - 3y + 4 = 0$$

$$2x - 3\left(\frac{22}{17}\right) + 4 = 0$$

$$2x - \frac{66}{17} + 4 = 0$$

$$2x = \frac{66}{17} - 4$$

$$2x = \frac{66 - 4(17)}{17}$$

$$2x = \frac{66-68}{17}$$

$$2x = \frac{-2}{17}$$

$$x = \frac{-2}{17 \times 2}$$

$$x = \frac{-1}{17}$$

Thus, the coordinate of point O is O $\left(\frac{-1}{17}, \frac{22}{17}\right)$.

Now, line OM is perpendicular to line AB.

When two lines are perpendicular, then the product of their slopes are equal to –1.

Slope of OM × slope of AB = –1

$$\text{Slope of OM} = \frac{-1}{\text{slope of } AB}$$

Equation of AB is $6x - 7y + 8 = 0$

$$6x + 8 = 7y$$
$$7y = 6x + 8$$
$$y = \frac{6x+8}{7}$$
$$y = \left(\frac{6}{7}\right)x + \frac{8}{7}$$

The above equation is of the form

$$y = mx + c$$

Where m is slope of line

$$\text{Slope of AB} = \frac{6}{7}$$

$$\text{Slope of OM} = \frac{-1}{\text{slope of AB}}$$
$$= \frac{-1}{\frac{6}{7}}$$
$$= \frac{-7}{6}$$

Now, we know that equation of line passing through (x_1, y_1) having slope m, is equation of line OM passing through O $\left(\frac{-1}{17}, \frac{22}{17}\right)$ having slope $\frac{-7}{6}$ is

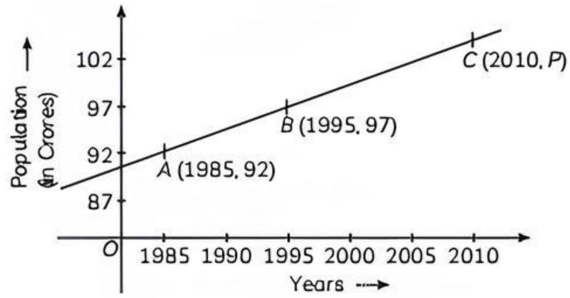
$$y - \frac{22}{17} = \frac{-7}{6}\left(x - \left(\frac{-1}{17}\right)\right)$$
$$y - \frac{22}{17} = \frac{-7}{6}\left(x + \frac{1}{17}\right)$$
$$y - \frac{22}{17} = \frac{-7}{6}x - \frac{7}{6} \times \frac{1}{17}$$
$$y - \frac{22}{17} = \frac{-7}{6}x - \frac{7}{6 \times 17}$$
$$y + \frac{7}{6}x = \frac{22}{17} - \frac{7}{6 \times 17}$$
$$\frac{6y+7x}{6} = \frac{22(6)-7}{6 \times 17}$$
$$\frac{6y+7x}{6} = \frac{132-7}{6 \times 17}$$
$$\frac{6y+7x}{6} = \frac{125}{6 \times 17}$$
$$17(6y + 7x) = 125$$
$$17(6y) + 17(7x) = 125$$
$$102y + 119x = 125$$

Hence, the required equation is

$$119x + 102y = 125$$

Example 2.15: Case Based:

India faces a major population crisis due to the growing population. If we were to estimate, we can say that almost 17% of the population of the world lives in India alone. India ranks second in the list of most populated countries. Population vs year graph given below:



Based on the above information, answer the following questions.

(A) Find the slope of line AB.

(B) The equation of line AB is:
 (a) $x + 2y = 1791$ (b) $x - 2y = 1801$
 (c) $x - 2y = 1791$ (d) $x - 2y + 1801 = 0$

(C) Find the population in the year 2010.

(D) The equation of line perpendicular to line AB and passing through (1995, 97) is:
 (a) $2x - y = 4087$ (b) $2x + y = 4087$
 (c) $2x + y = 1801$ (d) none of the above

(E) Assertion (A): The slope of the line $x + 7y = 0$ is $\frac{1}{7}$ and y-intercept is 0.

 Reason (R): The slope of the line $6x + 3y - 5 = 0$ is –2 and y-intercept is $\frac{5}{3}$.

 (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
 (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
 (c) (A) is true but (R) is false.
 (d) (A) is false but (R) is true.

Ans. **(A)** Slope of line

$$AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{97 - 92}{1995 - 1985}$$

$$= \frac{5}{10} = \frac{1}{2}$$

(B) *(b)* $x - 2y = 1801$

Explanation: Equation of line AB is

$$y - y_1 = m(x - x_1)$$

$$y - 92 = \frac{1}{2}(x - 1985)$$

$$2y - 184 = x - 1985$$

$$x - 2y = 1801$$

(C) Let the population in year 2010 is P.

Since, A, B, C are collinear

∴ Slope of AB = Slope of BC

$$\frac{97 - 92}{1995 - 1985} = \frac{P - 97}{2010 - 1995}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{P - 97}{15}$$

$$\Rightarrow \quad 7.5 = P - 97$$

$$\Rightarrow \quad P = 97 + 7.5 = 104.5 \text{ crores}$$

(D) *(b)* $2x + y = 4087$

Explanation: ∵ Slope of AB = $\frac{1}{2}$

Slope of line perpendicular to AB passing through (1995, 97) is

$$y - 97 = -2(x - 1995)$$

$$\Rightarrow \quad y - 97 = -2x + 3990$$

$$\Rightarrow \quad 2x + y = 4087$$

(E) *(d) (A) is false but (R) is true.*

Explanation: Given equation is $x + 7y = 0$

$$\Rightarrow \quad y = \frac{-x}{7} + 0$$

On comparing with $y = mx + c$, we get

Slope $(m) = \frac{-1}{7}$, y-intercept = 0

Now, consider $6x + 3y - 5 = 0$

$$\Rightarrow y = \frac{5}{3} - 2x$$

On comparing with $y = mx + c$, we get

Slope $(m) = -2$, y-intercept = $\frac{5}{3}$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. **A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y-intercept is:**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 1 (d) $\frac{4}{3}$

[NCERT Exemplar]

Ans. *(d)* $\frac{4}{3}$

Explanation: Slope of given line $3x + y = 3$ is given by

$$y = -3x + 3$$

On comparing it with $y = mx + c$, we get

$$m = -3$$

$$\therefore \quad \text{Slope of perpendicular line} = -\frac{1}{m} = \frac{1}{3}$$

Thus, the equation of the required line is

$$y - 2 = \frac{1}{3}(x - 2)$$

$$3y - 6 = x - 2$$

$$\Rightarrow \quad x - 3y + 4 = 0$$

For y – intercept, put $x = 0$.

$$0 - 3y + 4 = 0$$

$$\Rightarrow \quad y = \frac{4}{3},$$

which is y-intercept.

2. **Equation of the line passing through (1, 2)and parallel to the line $y = 3x - 1$ is:**

(a) $y + 2 = x + 1$ **(b)** $y + 2 = 3(x + 1)$

(c) $y - 2 = 3(x - 1)$ **(d)** $y - 2 = x - 1$

[NCERT Exemplar]

Ans. *(c)* $y - 2 = 3(x - 1)$

Explanation: Line is parallel to $y = 3x - 1$ and passes through the point (1, 2).

Hence, its slope is same as the given line i.e. $m = 3$

So, the equation of the line is: $y - 2 = 3(x - 1)$

3. **The equation line through point (0, 2) with slope –2 is:**

(a) $y + 2x = 2$ **(b)** $y - 2x = 2$

(c) $2x - y = 2$ **(d)** $d = 0$

Ans. *(a)* $y + 2x = 2$

Explanation: Here $x_1 = 0$, $y_1 = 2$ and $m = -2$, the equation will be

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -2(x - 0)$$
$$y - 2 = -2x$$
$$y + 2x = 2$$

4. **The equation of the straight line passing through the point (3, 2) and perpendicular to the line $y = x$ is:**

(a) $x - y = 5$ **(b) $x + y = 5$**

(c) $x + y = 1$ **(d) $x - y = 1$**

[NCERT Exemplar]

Ans. *(b) $x + y = 5$*

Explanation: Given that straight line passing through the point (3, 2) and perpendicular to the line $y = x$.

Let the equation of line 'L' is

$$y - y_1 = m(x - x_1)$$

Since L is passing through the point (3, 2).

$$\therefore \quad y - 2 = m(x - 3) \qquad \text{...(i)}$$

Now, given eq. is $y = x$

Since, the above equation is in $y = mx + b$ form.

So, the slope of this equation = 1

It is also given that line L and $y = x$ are perpendicular to each other.

We know that, when two lines are perpendicular, then

$$m_1 \times m_2 = -1$$
$$\therefore \quad m \times 1 = -1$$
$$\Rightarrow \quad m = -1$$

Putting the value of m in equation (i), we get

$$y - 2 = (-1)(x - 3)$$
$$\Rightarrow \quad y - 2 = -x + 3$$
$$\Rightarrow \quad x + y = 3 + 2$$
$$\Rightarrow \quad x + y = 5$$

5. **The equation of the line which has slope of 2 and cuts-off intercept of 6 on x-axis, is:**

(a) $2x - y + 12 = 0$ **(b) $2x + y + 12 = 0$**

(c) $-2x + y + 12 = 0$ **(d) $2x + y - 12 = 0$**

[Diksha]

Ans. *(c) $-2x + y + 12 = 0$*

Explanation: Let a and b be the intercepts of the line on X and Y-axes, respectively.

Equation of the line will be $\dfrac{x}{a} + \dfrac{y}{b} = 1$

$$\frac{x}{a} + \frac{y}{b} = 1$$
$$\frac{x}{6} + \frac{y}{b} = 1$$
$$bx + 6y = 6b$$
$$y = \frac{6b - bx}{6}$$
$$= \frac{-bx + 6b}{6}$$
$$y = \frac{-b}{6}x + b$$

[Comparing it with $y = mx + c$]

slope, $\quad m = \dfrac{-b}{6}$

But, according to the question $\dfrac{-b}{6} = 2$

Or, $\quad -b = 12$

$$b = -12$$

So, $\quad \dfrac{x}{6} + \dfrac{y}{-12} = 1$

$$\Rightarrow \quad \frac{x}{6} - \frac{y}{12} = 1$$
$$\frac{2x - y}{12} = 1$$
$$2x - y = 12$$
$$\Rightarrow \quad -2x + y + 12 = 0$$

6. **The equation of the line passing through (–2, 3) with slope –4 is:**

(a) $x + 4y - 10 = 0$ **(b) $y + 4x + 5 = 0$**

(c) $x + y - 1 = 0$ **(d) $3x + 4y - 6 = 0$**

[Diksha]

Ans. *(b) $y + 4x + 5 = 0$*

Explanation: We know that,

Equation of line passing through point (x_0, y_0) with slope m is

$$y - y_0 = m(x - x_0)$$

Here, slope = $m = -4$

Point $(x_0, y_0) = (-2, 3)$

Here, $\quad x_0 = -2$ and $y_0 = 3$

Putting the values

$$(y - 3) = -4(x - (-2))$$
$$(y - 3) = -4(x + 2)$$
$$y - 3 = -4x - 8$$
$$y + 4x - 3 + 8 = 0$$
$$y + 4x + 5 = 0$$

Thus, the required equation is $y + 4x + 5 = 0$.

7. **The equation of line passing through the points (1, 2) and (1, 3) is:**

(a) 0 **(b) not defined**

(c) $x - y = 2$ **(d) $x + y = 2$**

Ans. *(b) not defined*

Explanation: Here, $x_1 = 1$, $x_2 = 1$ and $y_1 = 2$, $y_2 = 3$.

So, equation of line will be

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - 2 = \frac{3 - 2}{1 - 1}(x - 1)$$
$$y - 2 = \frac{1}{0}(x - 1)$$

Slope = $\dfrac{1}{0}$ which is not defined.

8. A line cutting of intercept –3 on the y-axis and the tangent at angle to the x-axis is $\frac{3}{5}$ its equation is:

(a) $5y - 3x + 15 = 0$ (b) $3y - 5x + 15 = 0$

(c) $5y - 3x - 15 = 0$ (d) none of these

[NCERT Exemplar]

Ans. *(a) $5y - 3x + 15 = 0$*

Explanation: We know that,

Slope of a line, $m = \tan\theta$.

Given that, $\tan\theta = \frac{3}{5}$

$\Rightarrow$ Slope of line, $m = \frac{3}{5}$

Since, the lines cut off intercepts = –3 on y-axis then the line is passing through the point (0, –3).

So, the equation of line is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - (-3) = \frac{3}{5}(x - 0)$$

$$\Rightarrow \quad y + 3 = \frac{3}{5}x$$

$$\Rightarrow \quad 5y + 15 = 3x$$

$$\Rightarrow \quad 5y - 3x + 15 = 0$$

9. The distance of the point (4, –6) from the line $4x - 5y - 32 = 0$ is:

(a) $\frac{3}{7}$ (b) $\frac{14}{41}$

(c) $\frac{7}{5}$ (d) $\frac{14}{\sqrt{41}}$

Ans. *(d) $\frac{14}{\sqrt{41}}$*

Explanation: Given line is

$$4x - 5y - 32 = 0$$

Here, A = 4, B = –5 and C = –32

Given point is (4, –6).

So,

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

$$= \left|\frac{4 \times 4 + (-5)(-6) - 32}{\sqrt{16 + 25}}\right|$$

$$= \left|\frac{16 + 30 - 32}{\sqrt{41}}\right|$$

$$= \frac{14}{\sqrt{41}}$$

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

10. Assertion (A): Slope of x-axis is zero and slope of y-axis is not defined.

Reason (R): Slope of x-axis is defined and slope of y-axis is zero.

Ans. *(c) (A) is true but (R) is false.*

Explanation: slope of x-axis is zero and hence defined but slope of y-axis is not defined.

11. Assertion (A): The distance between the line $4x + 3y = 11$ and $8x + 6y = 15$ is $\frac{7}{10}$.

Reason (R): The distance between the line $ax + by = c_1$ and $ax + by = c_2$ is given by $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

Ans. *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: Given lines are

$$4x + 3y = 11$$

and $$2(4x + 3y) = 15$$

i.e., $$4x + 3y = \frac{15}{2}$$

Distance between them is

$$= \left|\frac{11 - \frac{15}{2}}{\sqrt{16 + 9}}\right|$$

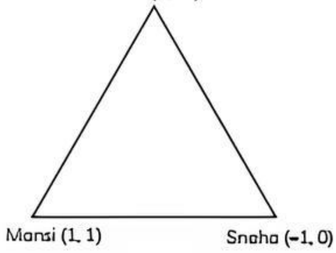
$$= \left|\frac{7}{10}\right| = \frac{7}{10}$$

CASE BASED Questions (CBQs)

[**4** & **5** marks]

Read the following passages and answer the questions that follow:

12. *Three girls Rani, Mansi, Sneha are talking to each other while maintaining a social distance due to covid-19. They are standing on vertices of a triangle, whose coordinates are given.*



Based on the above information answer the following questions.

(A) The equation of lines formed by Rani and Mansi is:

(a) $3x - y = 4$ (b) $3x + y = 4$
(c) $x - 3y = 4$ (d) $x + 3y = 4$

(B) Slope of equation of line formed by Rani and Sneha is:

(a) $\frac{2}{3}$ (b) $\frac{-3}{2}$
(c) $\frac{-2}{3}$ (d) $\frac{1}{3}$

(C) The equation of median of lines through Rani is:

(a) $5x + 4y = 2$ (b) $5x - 4y = 2$
(c) $4x - 5y = 1$ (d) none of these

(D) The equation of altitude through Mansi is:

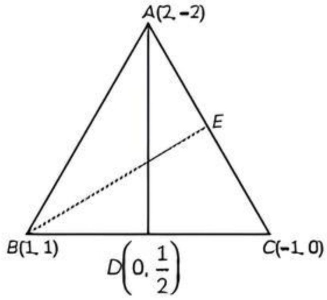
(a) $3x - 2y = 1$ (b) $2x + 3y = 5$
(c) $x + 2y = 3$ (d) none of these

(E) The equation of line passing through the Rani and parallel to line formed by Mansi and Sneha is:

(a) $x - 2y = 4$ (b) $x + 2y = 6$
(c) $x - 2y = 6$ (d) $2x + y = 4$

Ans. Let the point on Rani, Mansi and Sneha stand on a vertices of triangles be A, B, C.

$\therefore$ A(2, –2), B(1, 1), C(–1, 0)



(A) *(b)* $3x + y = 4$

Explanation: The equation of line AB is

$$y - 1 = \frac{-2-1}{2-1}(x-1)$$

$$\left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$$\Rightarrow \quad y - 1 = -3x + 3$$

$$\Rightarrow \quad 3x + y = 4$$

(B) *(c)* $-\frac{2}{3}$

Explanation: Slope of equation of line AC is

$$m = \frac{0+2}{-1-2} = \frac{2}{-3} = -\frac{2}{3}$$

(C) *(a)* $5x + 4y = 2$

Explanation: Let D be the mid-point of BC.

Coordinates of D are $\left(\frac{1-1}{2}, \frac{0+1}{2}\right) = \left(0, \frac{1}{2}\right)$

$\therefore$ Equation of AD is $y + 2 = \frac{\frac{1}{2}+2}{0-2}(x-2)$

$$\Rightarrow \quad y + 2 = \frac{-5}{4}(x-2)$$

$$\Rightarrow \quad 4y + 8 = -5x + 10$$

$$\Rightarrow \quad 5x + 4y = 2$$

(D) *(a)* $3x - 2y = 1$

Explanation: Slope of AC $= \frac{-2}{3}$

$\therefore$ Slope of BE $= \frac{3}{2}$ [$\because$ BE $\perp$ AC]

Equation of altitude through B is

$$y - 1 = \frac{3}{2}(x-1)$$

$$\Rightarrow \quad 3x - 2y = 1$$

(E) *(c)* $x - 2y = 6$

Explanation: Slope of line BC $= \frac{0-1}{-1-1} = \frac{1}{2}$

Equation of line passing through A and parallel to BC is

$$y + 2 = \frac{1}{2}(x-2)$$

$$\Rightarrow \quad 2y + 4 = x - 2$$

$$\Rightarrow \quad x - 2y = 6$$

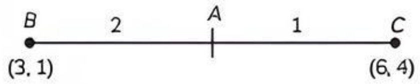
13. *A triangular park has two of its vertices as B(–4, 1) and C(2, 11). The third vertex A is a point dividing the line joining the points (3, 1) and (6, 4) in the ratio 2 : 1.*

(A) Find the coordinates of third vertex A.

(B) Find the equation of line passing through B and C.

(C) Find the equations of the sides of a triangle whose vertices are A(–1, 8), B(4, –2) and C(–5, –3).

Ans. (A)



Coordinates of

$$A = \left(\frac{2\times6+1\times3}{2+1}, \frac{2\times4+1\times1}{2+1}\right)$$

i. e., (5, 3)

(B) Equation of line through B(–4, 1)and C(2, 11) is

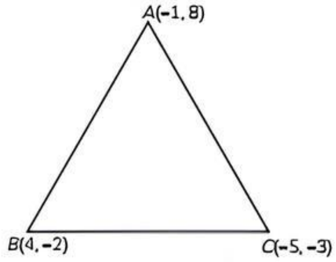
$$y-1=\frac{11-1}{2+4}(x+4)$$

$\Rightarrow \quad y-1=\frac{5}{3}(x+4)$

$\Rightarrow \quad 3y - 3 = 5x + 20$

$\Rightarrow \quad 5x - 3y + 23 = 0$

(C) Here, we use two points form to find the equation of sides.



Equation of AB is $y - 8 = \frac{-2-8}{4+1}(x+1)$

$\Rightarrow \quad 5(y - 8) + 10(x + 1) = 0$

$\Rightarrow \quad 10x + 5y - 30 = 0$

$\Rightarrow \quad 2x + y - 6 = 0$

[dividing both sides by 5]

Equation of BC is

$$y + 2 = \frac{-3+2}{-5-4}(x-4)$$

$\Rightarrow \quad -9(y + 2) + (x - 4) = 0$

$\Rightarrow \quad x - 9y - 22 = 0$

Equation of AC is $y-8=\frac{-3-8}{-5+1}(x+1)$

$\Rightarrow \quad -4(y - 8) + 11(x + 1) = 0$

$\Rightarrow \quad 11x - 4y + 43 = 0$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

14. Find the equation of a line with slope (–3) and cutting off an intercept of 3 units on the negative *y*-axis.

Ans. Here, $m = -3$ and $c = -3$

So equation of line is

$\therefore \quad y = mx + c$

$y = -3x - 3$

Or $3x + y + 3 = 0$

15. Determine the equation line through the point (–2, –1) and parallel to *x*-axis.

Ans. Here, $m = 0$

$x_1 = -2, y_1 = -1$

So, the equation of line is

$y - y_1 = m(x - x_1)$

$y + 1 = 0(x + 2)$

$y + 1 = 0$

16. Find the equation of the line, which makes intercepts of 2 and –3 on the *x*-axis and *y*-axis respectively.

Ans. Given that *a* line makes intercepts 2 and –3 on the *x* and *y*-axis respectively.

Then, $a = 2$ and $b = -3$

Hence, the equation of line intercepts form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{2}-\frac{y}{3}=1$$

17. Write the equation of the line for which $\tan\theta = \frac{1}{2}$, where θ is the inclination of the line and *x*-intercept is 4.

Ans. Given that, $\tan\theta = \frac{1}{2}$, where θ is the inclination of the line.

Then, the slope of a line, $m = \tan\theta = \frac{1}{2}$

Also, given that *x*–intercept is $d = 4$.

Hence, the equation of the line in slope-intercept form is

$$y = m(x - d)$$

$$\Rightarrow \quad y = \frac{1}{2}(x-4)$$

18. Find the equation of the line passing through the point (1, 2) and perpendicular to the line $x + y + 1 = 0$. [NCERT Exemplar]

Ans. Given that line passing through the point (1, 2)

And perpendicular to the line $x + y + 1 = 0$.

Let the equation of line 'L' is

$$x - y + k = 0 \quad ...(i)$$

Since, L is passing through the point (1, 2)

$$\therefore \quad 1 - 2 + k = 0$$

$$\Rightarrow \quad k = 1$$

Putting the value of k in equation (i), we get

$$x - y + 1 = 0$$

Or $\quad y - x - 1 = 0.$

19. Find the equation of the line through (2, 3) so that the segment of the line intercepted between the axes is bisected at this point. [Diksha]

Ans. Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$ which meets the x and y axes at A(a, 0) and B(0, b) respectively.

The coordinate of the mid-point of AB are $\left(\frac{a}{2},\frac{b}{2}\right)$. It is given that the point (2, 3) bisects AB

$$\frac{a}{2}=2 \quad \text{and} \quad \frac{b}{2}=3$$

$$a = 4 \quad \text{and} \quad b = 6$$

On putting, $a = 4$ and $b = 6$ in $\frac{x}{a}+\frac{y}{b}=1$, we get

$$\frac{x}{4}+\frac{y}{6}=1 \quad \text{or} \quad 3x + 2y = 12$$

Hence, the equation is $3x + 2y = 12$.

20. Find the equation of the line intersecting the y-axis at a distance of 4 units above the origin and making an angle of 30° with the positive direction of the x-axis.

Ans. Then, the slope of the line is $m = \tan 30° = \frac{1}{\sqrt{3}}$.

Also, given that the line intersects the y-axis at a distance of 4 units above the origin.

So, the y–intercept is $c = 4$.

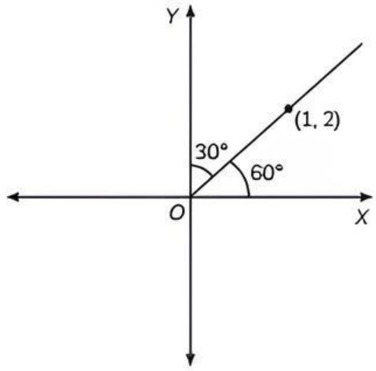
Hence, the equation of the line in slope-intercept form is

$$y = mx + c$$

$$\Rightarrow \quad y = \frac{x}{\sqrt{3}}+4$$

21. Find the equation of lines passing through (1, 2) and making angle 30° with y-axis [NCERT Exemplar]

Ans. Given that line passing through (1, 2) making an angle 30° with y-axis.



Angle made by the line with x-axis is $(90° - 30°)$
$= 60°$

$\therefore$ Slope of the line, $m = \tan 60° = \sqrt{3}$

So, the equation of the line passing through the point (x_1, y_1) and having slope 'm' is

$$y - y_1 = m(x - x_1)$$

Here, $\quad (x_1, y_1) = (1, 2)$

and $\quad m = \sqrt{3}$

$$\Rightarrow \quad y - 2 = \sqrt{3}(x-1)$$

$$\Rightarrow \quad y - 2 = \sqrt{3}x - \sqrt{3}$$

$$\Rightarrow \quad y - \sqrt{3}x + \sqrt{3} - 2 = 0$$

22. A straight line moves so that the sum of the reciprocals of its intercepts made on axes is constant. Show that the line passes through a fixed point. [NCERT Exemplar]

Ans. We know that intercepts form of a straight line is

$$\frac{x}{a}+\frac{y}{b}=1$$

where a and b are the intercepts on the axes.

Given that $\frac{1}{a}+\frac{1}{b}=\frac{1}{k}$ (say)

On cross multiplication we get

$$\Rightarrow \quad \frac{k}{a}+\frac{k}{b}=1$$

This shows that the line is passing through the fixed point (k, k).

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

23. Find the equation of the straight line which passes through the point (1, –2) and cuts off equal intercepts from axes. **[NCERT Exemplar]**

Ans. The equation of line in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

where, a and b are the intercepts on the axis

Given that, $a = b$

$$\Rightarrow \quad \frac{x}{a}+\frac{y}{a}=1$$

The above equation can be written as

$$\Rightarrow \quad \frac{x+y}{a}=1$$

On cross multiplication, we get

$$\Rightarrow \quad x + y = a \quad ...(i)$$

If equation (i) passes through the point (1, –2), we get

$$x = 1 \text{ and } y = -2$$
$$1 + (-2) = a$$
$$\Rightarrow \quad 1 - 2 = a$$
$$\Rightarrow \quad a = -1$$

Putting the value of a in equation (i), we get

$$x + y = -1$$
$$\Rightarrow \quad x + y + 1 = 0$$

Hence, the equation of straight line is $x + y + 1 = 0$ which passes through the point (1, –2).

24. A straight line cuts intercepts from the axes of the coordinate, the sum of whose reciprocals is a constant. Show that it always passes through a fixed point.

Ans. Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$

Its intercepts on x and y axes are a and b respectively. It is given that

$$\frac{1}{a}+\frac{1}{b} = \text{constant} = p \text{ (say)}$$

$$\therefore \quad \frac{1}{pa}+\frac{1}{pb}=1$$

$$\Rightarrow \quad \frac{\frac{1}{p}}{a}+\frac{\frac{1}{p}}{b}=1$$

$\Rightarrow \left(\frac{1}{p},\frac{1}{p}\right)$ satisfies the equation $\frac{x}{a}+\frac{y}{b}=1$

Hence, the line (i) passes through the fixed point $\left(\frac{1}{p},\frac{1}{p}\right)$.

25. Find the equation of the line for which $\tan\theta = \frac{1}{2}$, where θ is the inclination of the line and
(A) x-intercept equal to 4.
(B) y-intercept is $-\frac{3}{2}$. **[NCERT Exemplar]**

Ans. (A) Clearly, the line passes through (4, 0) and has slope = $\frac{1}{2}$.

So, the equation of the line is

$$y - 0 = \frac{1}{2}(x-4)$$

[putting $x_1 = 4$, $y_1 = 0$ and $m = \frac{1}{2}$ in $y - y_1 = m(x - x_1)$]

$$\Rightarrow \quad x - 2y - 4 = 0$$

(B) The line passes through $\left(0,-\frac{3}{2}\right)$ and has slope $= \frac{1}{2}$.

So, it equation is

$$y - \frac{3}{2} = \frac{1}{2}(x - 0)$$

[putting $x_1 = 0$, $y_1 = -\frac{3}{2}$ and $m = \frac{1}{2}$ in $y - y_1 = m(x - x_1)$]

$$\Rightarrow \quad 2y + 3 = x \text{ or, } x - 2y + 3 = 0$$

26. Find the equations of the straight lines which pass through the origin and trisect the intercept of the line $9x + 12y = 36$ between the axes.

Ans. The equation of the given line is

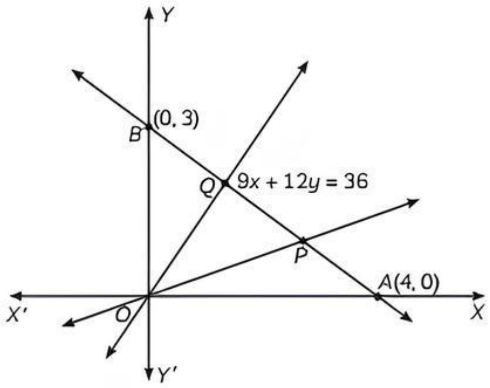
$$9x + 12y = 36 \text{ or, } \frac{x}{4}+\frac{y}{3}=1$$

It cuts the coordinate axes at A(4, 0) and B(0, 3).

The portion AB of the given line intercepted between the axes is trisected by points P and Q.

$$\therefore \quad \frac{AP}{AB}=\frac{1}{2} \text{ and } \frac{AQ}{QB}=\frac{2}{1}$$

$\Rightarrow$ P and Q divide AB internally in the ratio 1 : 2 and 2 : 1 respectively.



So, coordinates P and Q are

$$P\left(\frac{1\times0+2\times4}{1+2}, \frac{1\times3+2\times0}{1+2}\right)=P\left(\frac{8}{3},1\right),$$

$$Q\left(\frac{2\times0+1\times4}{2+1}, \frac{2\times3+1\times0}{2+1}\right)=Q\left(\frac{4}{3},2\right)$$

Hence, the equation of OQ is

$$y-0=\frac{2-0}{\frac{4}{3}-0}(x-0) \text{ or, } 3x-2y=0$$

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

27. Find the equation of the lines which passes through the point (3, 4) and cuts off intercepts from the coordinates axes such that their sum is 14. [NCERT Exemplar]

Ans. The equation of line in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \quad \text{...(i)}$$

where a and b are the intercepts on the axis.

Given that $a + b = 14$

The above equation can be written as

$\Rightarrow$ $b = 14 - a$

Substituting the value of a and b in equation (i) we get

So, equation of line is

$$\frac{x}{a}+\frac{y}{14-a}=1$$

Taking LCM

$$\Rightarrow \quad \frac{x(14-a)+ay}{(a)(14-a)}=1$$

$$\Rightarrow \quad 14x - ax + ay = 14a - a^2 \quad \text{...(ii)}$$

If equation (ii) passes through the point (3, 4) then

$$14(3) - a(3) + a(4) = 14a - a^2$$

$$\Rightarrow 42 - 3a + 4a - 14a + a^2 = 0$$

$$\Rightarrow a^2 - 13a + 42 = 0$$

$$\Rightarrow a^2 - 7a - 6a + 42 = 0$$

$$\Rightarrow a(a - 7) - 6(a - 7) = 0$$

$$\Rightarrow (a - 6)(a - 7) = 0$$

$$\Rightarrow a - 6 = 0 \text{ or } a - 7 = 0$$

$$\Rightarrow a = 6 \text{ or } a = 7$$

If $a = 6$, then

$$6 + b = 14$$

$$\Rightarrow b = 14 - 6 = 8$$

If $a = 7$, then

$$7 + b = 14$$

$$\Rightarrow b = 14 - 7$$

$$\Rightarrow b = 7$$

If $a = 6$ and $b = 8$, then equation of line is

$$\frac{x}{6}+\frac{y}{8}=1$$

$$\Rightarrow \frac{4x+3y}{24}=1$$

$$\Rightarrow 4x + 3y = 24$$

If $a = 7$ and $b = 7$, then equation of line is

$$\frac{x}{7}+\frac{y}{7}=1$$

$$\Rightarrow x + y = 7$$

28. Find the equation of the line passing through the point (5, 2) and perpendicular to the line joining the points (2, 3) and (3, −1). [NCERT Exemplar]

Ans. Given points are A(5 ,2), B(2, 3) and C(3, −1).

Firstly, we find the slope of the line joining the points (2, 3) and (3, −1).

$$\text{Slope of the line joining two points} = \frac{y_2-y_1}{x_2-x_1}$$

$$\therefore \quad m_{BC}=\frac{-1-3}{3-2}=-\frac{4}{1}=-4$$

It is given that line passing through the point (5, 2) is perpendicular to BC

$$m_1 m_2 = -1$$

$$\Rightarrow -4 \times m_2 = -1$$

$\Rightarrow \qquad m_2 = \frac{1}{4}$

Therefore, slope of the required line = $\frac{1}{4}$

Now, we have to find the equation of line passing through point (5, 2)

Equation of line: $y - y_1 = m(x - x_1)$

$\Rightarrow \qquad y - 2 = \frac{1}{4}(x-5)$

$\Rightarrow \qquad 4y - 8 = x - 5$

$\Rightarrow \qquad x - 5 - 4y + 8 = 0$

$\Rightarrow \qquad x - 4y + 3 = 0$

Hence, the equation of line passing through the point (5, 2) is $x - 4y + 3 = 0$.

29. Find the equations of the altitudes of the triangle whose vertices are A(7, –1), B(–2, 8) and C(1,2).

Ans. Let AD, BE and CF be three altitude of triangle ABC and let m_1, m_2 and m_3 be the Slopes of AD, BE and CF, respectively. Then,

$AD \perp BC$

$\Rightarrow$ Slope of AD × Slope of BC = –1

$\Rightarrow \qquad m_1 \times \left(\frac{2-8}{1+2}\right) = -1$

$\Rightarrow \qquad m_1 = \frac{1}{2}$

$BE \perp AC$

$\Rightarrow$ Slope of BE × Slope of AC = –1

$\Rightarrow \qquad m_2 \times \left(\frac{-1-2}{7-1}\right) = -1$

$\Rightarrow \qquad m_2 = 2$

and $CF \perp AB$

$\Rightarrow$ Slope of CF slope of AB = –1

$\Rightarrow \qquad m_3 \times \left(\frac{-1-8}{7+2}\right) = -1$

$\Rightarrow \qquad m_3 = 1$

Since, AD passes through A(7, –1) and has slope

$m_1 = \frac{1}{2}$

So, its equation is

$y + 1 = \frac{1}{2}(x-7)$

$\Rightarrow \qquad x - 2y - 9 = 0$

Similarly, equation of BE is

$y - 8 = 2(x + 2)$

$\Rightarrow \qquad 2x - y + 12 = 0$

equation of CF is $(x - 1)$

$\Rightarrow \qquad x + y + 1 = 0$

30. A line passing through the point A(3, 0) makes 30° angle with the positive direction of x-axis. If this line is rotated through an angle 15° in a clockwise direction. Find its equation in new position.

Ans. Let AB be in the given line and AC be its new position. Clearly, AC makes an angle of 15° with the positive direction of X-axis.

$\therefore \qquad m$ = Slope of AC = tan 15°

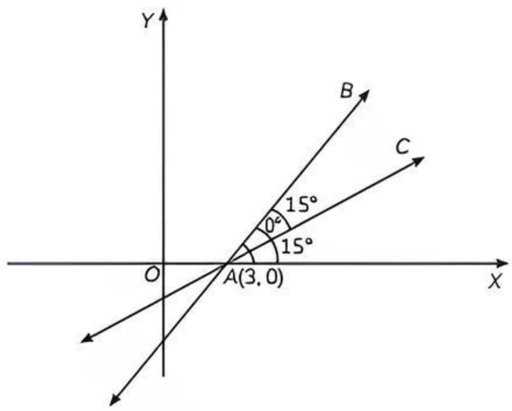
$\Rightarrow \qquad m = \tan(45° - 30°)$

$\Rightarrow \qquad m = \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$\Rightarrow \qquad m = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\Rightarrow \qquad m = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$= \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$



Clearly, AC passes through A(3, 0) and has slope $m = 2-\sqrt{3}$.

So, its equation is

$y - 0 = (2-\sqrt{3})(x-3)$

or, $(2-\sqrt{3})x - y - 3(2-\sqrt{3}) = 0$

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

31. In what direction should a line be drawn through the point (1, 2) so that its point of intersection with the line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point?

[NCERT Exemplar]

Ans. Let the given line $x + y = 4$ and the required line 'l' intersect at B (a, b)

Slope of line 'l' is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b-2}{a-1}$$

And we also know that, $m = \tan\theta$

$$\therefore \quad \tan\theta = \frac{b-2}{a-1} \quad \text{...(i)}$$

Given that AB $= \frac{\sqrt{6}}{3}$

So, by distance formula for point A(1, 2) and B(a, b), we get

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \quad \frac{\sqrt{6}}{3} = \sqrt{(a-1)^2 + (b-2)^2}$$

On squaring both the sides, we get

$$\Rightarrow \quad \frac{6}{9} = (a-1)^2 + (b-2)^2$$

Using $(a - b)^2$ formula we get

$$\Rightarrow \quad \frac{2}{3} = a^2 + 1 - 2a + b^2 + 4 - 4b$$

On cross multiplication, we get

$$\Rightarrow \quad 2 = 3a^2 + 3 - 6a + 3b^2 + 12 - 12b$$

$$\Rightarrow \quad 2 = 3a^2 + 3b^2 - 6a - 12b + 15$$

$$\Rightarrow \quad 3a^2 + 3b^2 - 6a - 12b + 13 = 0 \quad \text{...(ii)}$$

Point B(a, b) also satisfies the equation $x + y = 4$.

$$\therefore \quad a + b = 4$$

$$\Rightarrow \quad b = 4 - a \quad \text{...(iii)}$$

Putting the value of b in equation (ii), we get

$$3a^2 + 3(4-a)^2 - 6a - 12(4-a) + 13 = 0$$

Computing and simplifying we get

$$\Rightarrow \quad 3a^2 + 3(16 + a^2 - 8a) - 6a - 48 + 12a + 13 = 0$$

$$\Rightarrow \quad 3a^2 + 48 + 3a^2 - 24a - 6a - 48 + 12a + 13 = 0$$

$$\Rightarrow \quad 6a^2 - 18a + 13 = 0$$

Now, we solve the above equation by using the formula

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$a = \frac{-(-18) \pm \sqrt{(-18)^2 - 4 \times 6 \times 13}}{2 \times 6}$$

$$a = \frac{18 \pm \sqrt{324 - 312}}{12}$$

$$a = \frac{18 \pm \sqrt{12}}{12}$$

$$a = \frac{9 \pm \sqrt{3}}{6}$$

$$a = \frac{\sqrt{3}(3\sqrt{3} \pm 1)}{\sqrt{3}(2\sqrt{3})}$$

$$a = \frac{3\sqrt{3} \pm 1}{2\sqrt{3}}$$

$$\Rightarrow \quad a = \frac{3\sqrt{3}+1}{2\sqrt{3}} \text{ or } a = \frac{3\sqrt{3}-1}{2\sqrt{3}})$$

Putting the value of a in equation (iii), we get

$$b = 4 - \frac{3\sqrt{3}+1}{2\sqrt{3}}$$

Taking LCM and simplifying we get

$$\Rightarrow \quad b = \frac{8\sqrt{3} - 3\sqrt{3} \pm 1}{2\sqrt{3}}$$

$$\Rightarrow \quad b = \frac{5\sqrt{3} \pm 1}{2\sqrt{3}}$$

$$\Rightarrow \quad b = \frac{5\sqrt{3}+1}{2\sqrt{3}} \text{ or } b = \frac{5\sqrt{3}-1}{2\sqrt{3}}$$

Now, putting the value of a and b in equation (i), we get

$$\tan\theta = \frac{b-2}{a-1}$$

$$\Rightarrow \quad \tan\theta = \frac{\frac{5\sqrt{3} \pm 1}{2\sqrt{3}} - 2}{\frac{3\sqrt{3} \pm 1}{2\sqrt{3}} - 1}$$

Taking LCM and simplifying we get

$$\Rightarrow \quad \tan\theta = \frac{\frac{5\sqrt{3} \pm 1 - 4\sqrt{3}}{2\sqrt{3}}}{\frac{3\sqrt{3} \pm 1 - 2\sqrt{3}}{2\sqrt{3}}}$$

$\Rightarrow \quad \tan\theta = \dfrac{\sqrt{3}\pm 1}{\sqrt{3}\pm 1}$

$\Rightarrow \quad \tan\theta = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ ...(iv)

$\tan\theta = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ ...(v)

We solve the equation (i) to get the value of θ, we get

We know that,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

If $x = \sqrt{3}$ and $y = 1$

$$= \tan^{-1}\left(\frac{\sqrt{3}-1}{1+(\sqrt{3})1}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}-1}{1+\sqrt{3}}\right)$$

We have,

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$$

$$\theta = \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$$

$$\theta = \tan^{-1}\tan 60° - \tan^{-1}\tan 45°$$

$$\theta = 60° - 45°$$

$$\theta = 15°$$

Now, we solve the equation (v)

We know that,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

If $x = \sqrt{3}$ and $y = 1$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}+1}{1-(\sqrt{3})(1)}\right)$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}+1}{1-\sqrt{3}}\right)$$

We have,

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

$$\theta = \tan^{-1}(\sqrt{3}) + \tan^{-1}(1)$$

$$\theta = \tan^{-1}\tan 60° + \tan^{-1}\tan 45°$$

$$\theta = 60° + 45°$$

$$\theta = 105°$$

32. Find the equation of the line which passes through the point (–5, 4) and the portion of the line intercepted between the axes is divided internally in the ratio 5 : 4 by this point.

Ans. Let AB be a line passing through a point (–5, 4) and meets x-axis at A(a, 0) and y-axis at B(0, b).

Using the section formula for internal division, we have

$$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \quad \text{...(i)}$$

Here, $m_1 = 5, m_2 = 4$

$(x_1, y_1) = (a, 0)$ and $(x_2, y_2) = (0, b)$

Substituting the above values in the above formula, we get

$$\Rightarrow \quad x = \frac{5(0)+4(a)}{5+4}, y = \frac{5(b)+4(0)}{5+4}$$

$$\Rightarrow \quad -5 = \frac{4a}{9}, 4 = \frac{5b}{9}$$

$$\Rightarrow \quad -45 = 4a \text{ or } 36 = 5b$$

$$\Rightarrow \quad -a = -\frac{45}{4} \text{ or } b = \frac{36}{5}$$

We know that intercept of the line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Substituting the value of a and b in above equation, we get

$$\frac{x}{-\frac{45}{4}} + \frac{y}{\frac{36}{5}} = 1$$

On simplification, we get

$$\Rightarrow \quad \frac{-4x}{45} + \frac{5y}{36} = 1$$

Taking LCM

$$\Rightarrow \quad \frac{-4x\times 4+5\times 5y}{9\times 5\times 4} = 1$$

On cross multiplication we get

$$\Rightarrow \quad -16x + 25y = 180$$

which is the required equation of the line.